EE 435

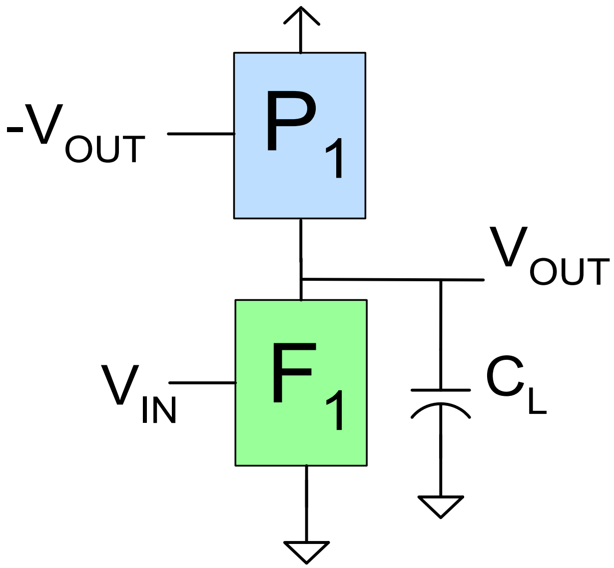
Lecture 20

Gain Enhancement with Regenerative Feedback

Linearity in Operational Amplifiers

-- The differential pairs

Gain Enhancement with Regenerative Feedback



$$A_{V0} = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}}$$

$$A_{V0} = \frac{g_{mF1}}{g_{oF1} + g_{oP1} - g_{mP1}}$$

$$BW = \frac{g_{oF1} + g_{oP1} - g_{mP1}}{C_L}$$

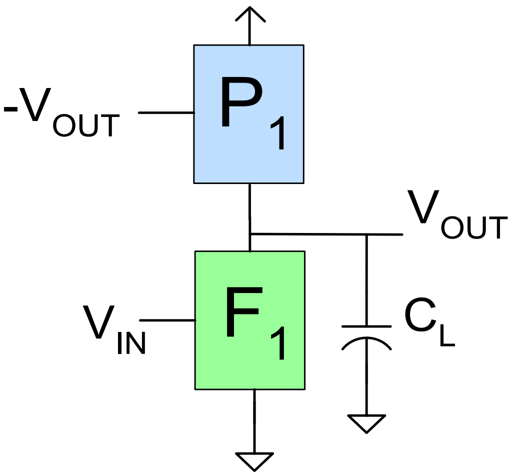
$$GB = \frac{g_{mF1}}{C_L}$$

The gain can be made arbitrarily large by selecting g_{mP1} appropriately

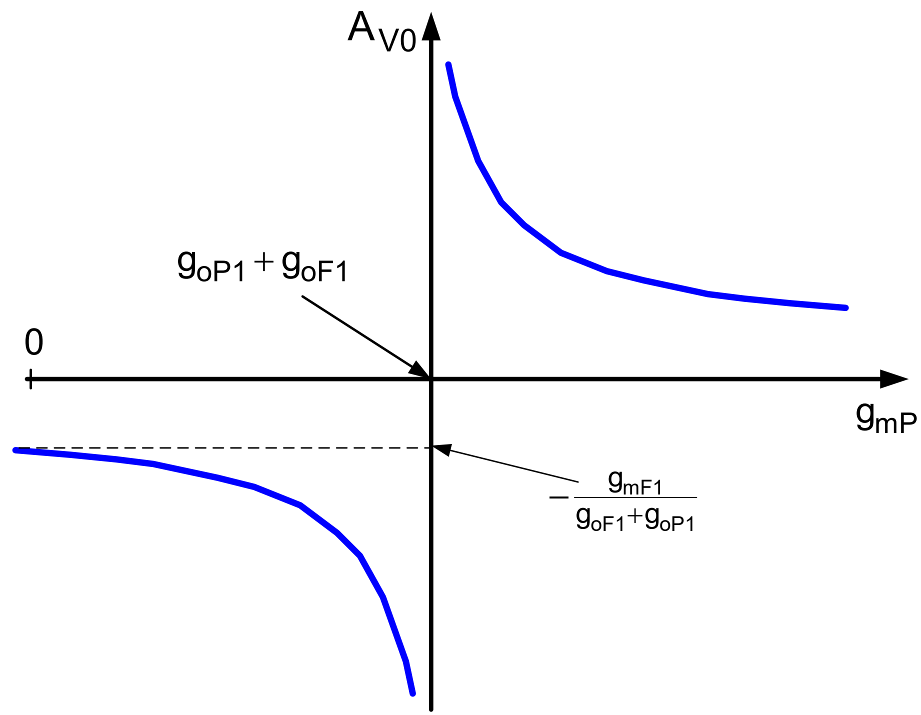
The GB does not degrade !

But - can we easily build circuits with this property?

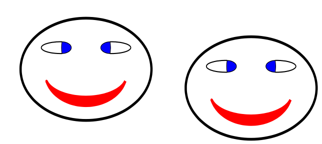
Gain Enhancement with Regenerative Feedback



$$A_{V0}(s) = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}}$$



If $g_{mP1} = g_{oF1} + g_{oP1}$, the dc gain will become infinite !!

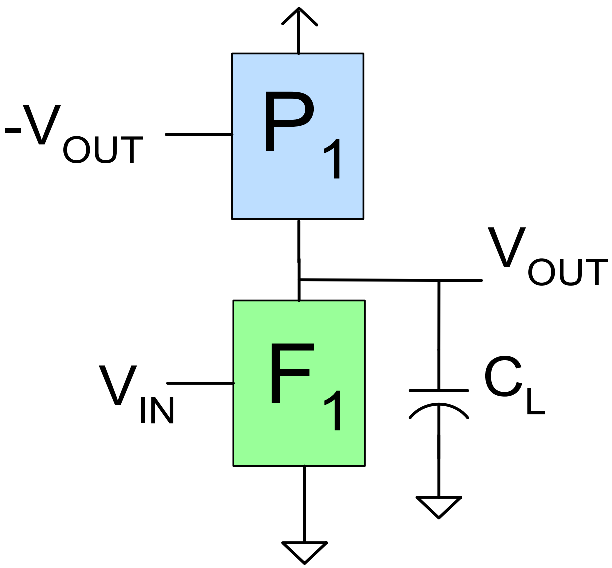


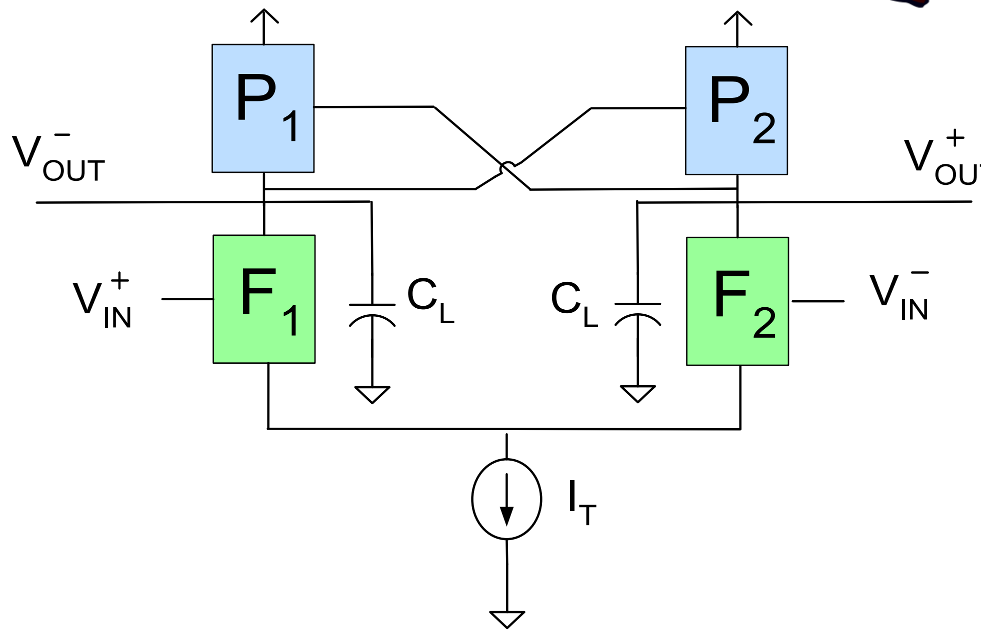
Term this “gain reversing” when dc gain changes sign with pole

Gain Enhancement with Regenerative Feedback

- But - can we easily build circuits with this property?

Review from last lecture.

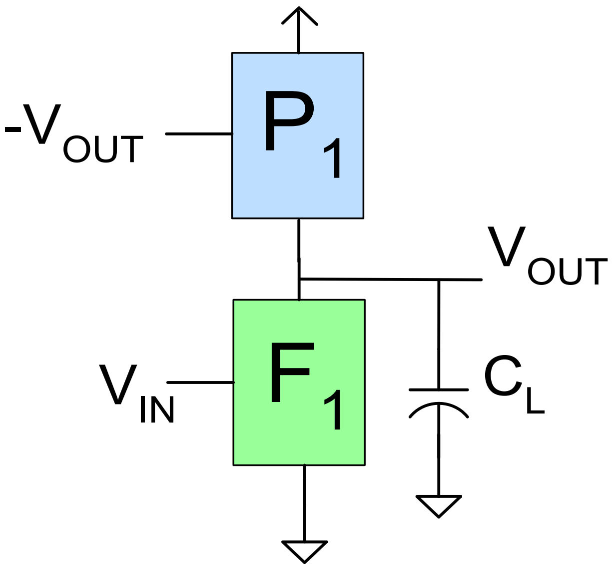




- But – the inverting amplifier may be more difficult to build than the op amp itself!

YES – simply by cross-coupling the outputs in a fully differential structure

Gain Enhancement with Regenerative Feedback



$$A_{0L} = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}}$$

Observe:

$$A_{V0} = \frac{A_0\tilde{p}}{s+\tilde{p}}$$

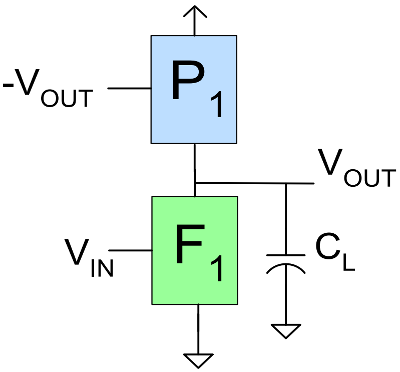
So pole: $p = -\tilde{p}$

Observe for this amplifier:

$$A_0 \begin{cases} < 0 & \text{if } \tilde{p} > 0 \\ > 0 & \text{if } \tilde{p} < 0 \end{cases}$$

Thus dc gain reversing occurs when pole changes from positive to negative

Gain Enhancement with Regenerative Feedback



It will be shown that a feedback amplifier with dc gain reversing with pole is usually stable even if the open-loop Op amp is unstable!

$$A_{OL} = \frac{A_{OL}\tilde{p}}{s+\tilde{p}}$$

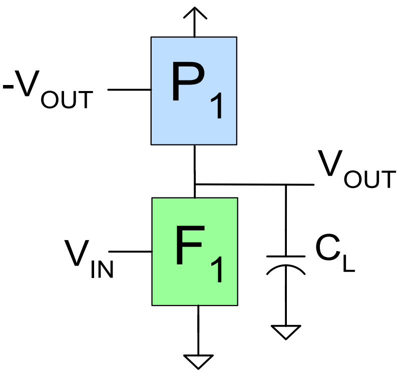
Assume standard feedback model $$A_{FB} = \frac{A_{OL}}{1+A_{OL}\beta}$$

$$A_{FB} = \frac{A_{OL}\tilde{p}}{s+\tilde{p}\left(1+\beta A_{OL}\right)}$$

Due to “gain reversal”

$$p_{FB} = \begin{cases} p_1\left(1+\beta A_{V0}\right) & \text{for } p_1 < 0 \\ p_1\left(1-\left|\beta A_{V0}\right|\right) & \text{for } p_1 > 0 \end{cases}$$

Gain Enhancement with Regenerative Feedback

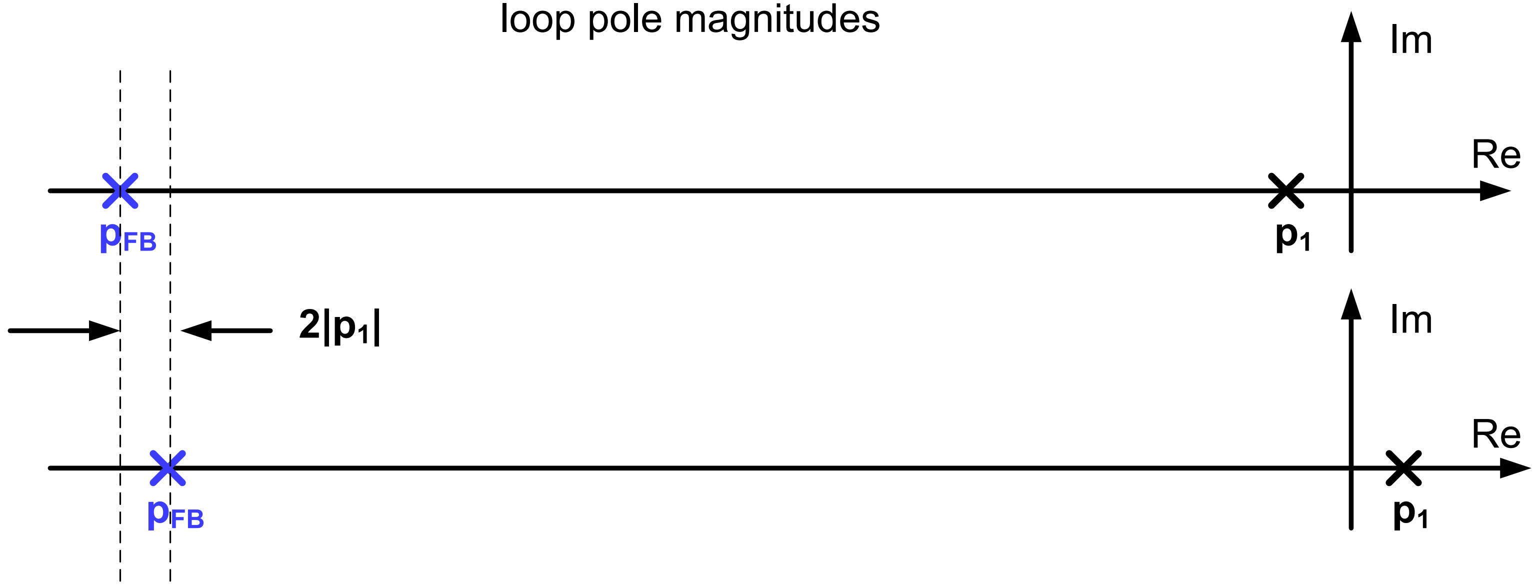


It will be shown that a feedback amplifier with dc gain reversing with pole is usually stable even if the open-loop Op amp is unstable!

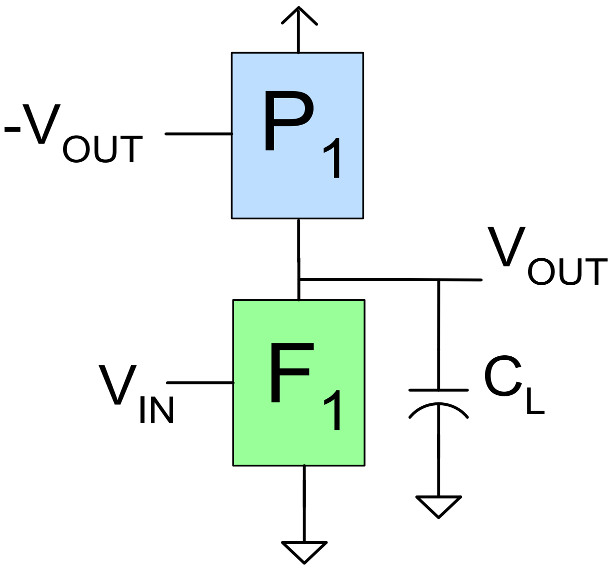
How?

$$p_{FB} = \begin{cases} p_1\left(1+\beta A_{V0}\right) & \text{for } p_1 < 0 \\ p_1\left(1-\left|\beta A_{V0}\right|\right) & \text{for } p_1 > 0 \end{cases}$$

Open-Loop and Closed-Loop Pole Plot for equal open-loop pole magnitudes



Gain Enhancement with Regenerative Feedback



$$A_{V0} = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}}$$

$$p = \frac{-g_{oF1} - g_{oP1} + g_{mP1}}{C_L}$$

If $g_{mP1} > g_{oF1} + g_{oP1}$, the pole will be in the RHP !!

The feedback performance can actually be enhanced if the open-loop amplifier with gain reversal is unstable

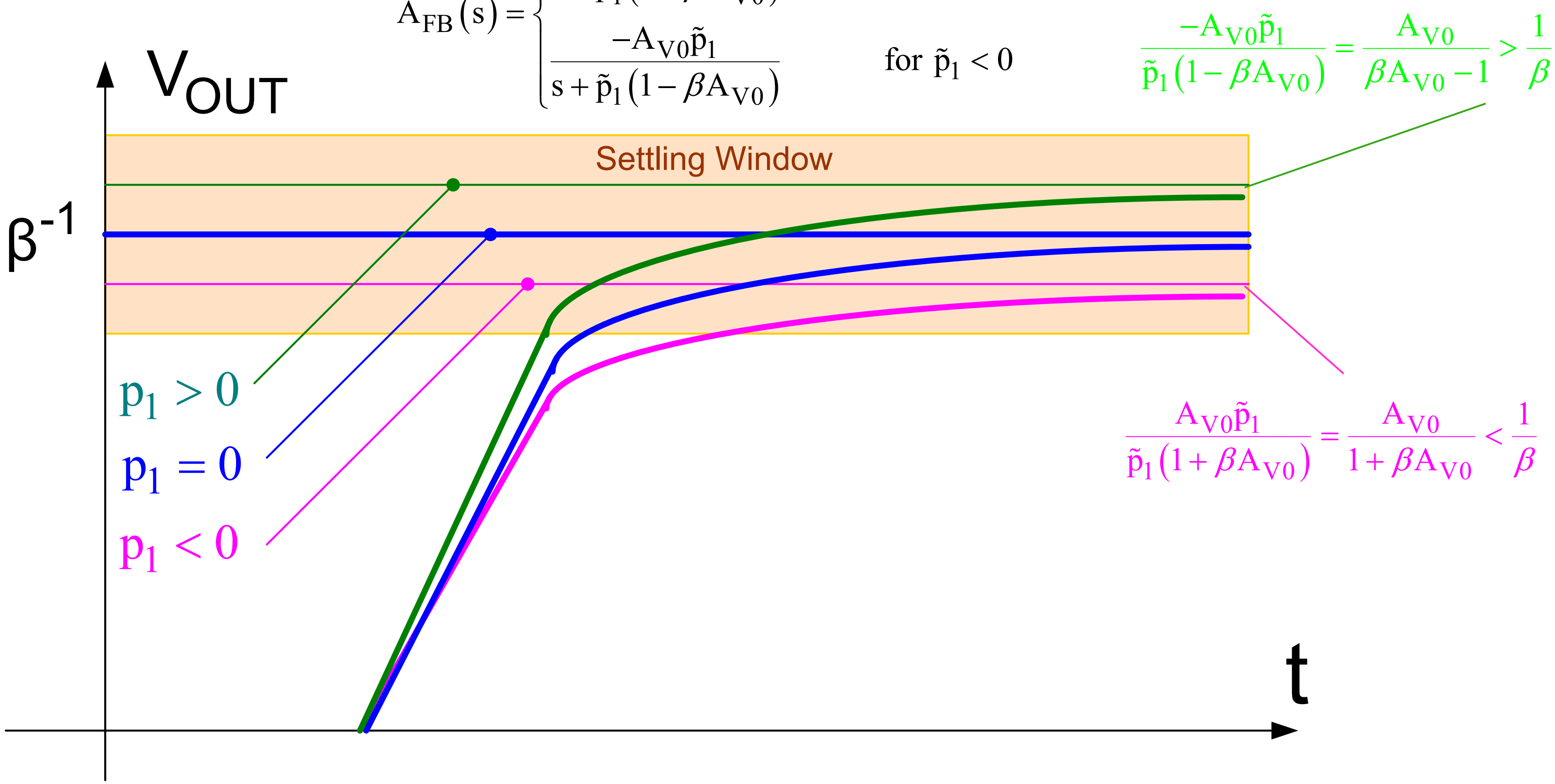
Why?

Gain Enhancement with Regenerative Feedback

The feedback performance can actually b enhanced if the open-loop amplifier is unstable

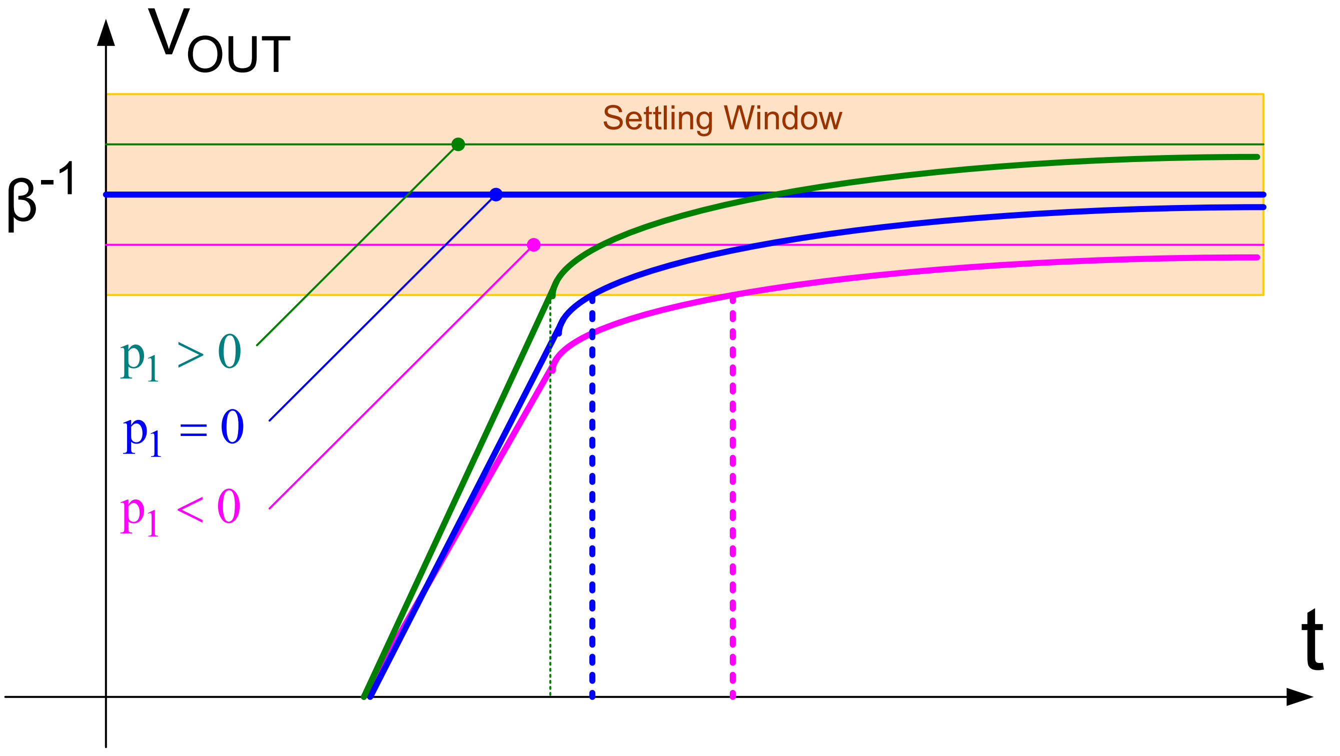
Why?

$$A_{FB}(s) = \begin{cases} \dfrac{A_{V0}\tilde{p}_1}{s+\tilde{p}_1\left(1+\beta A_{V0}\right)} & \text{for } \tilde{p}_1 > 0 \\ \dfrac{-A_{V0}\tilde{p}_1}{s+\tilde{p}_1\left(1-\beta A_{V0}\right)} & \text{for } \tilde{p}_1 < 0 \end{cases}$$



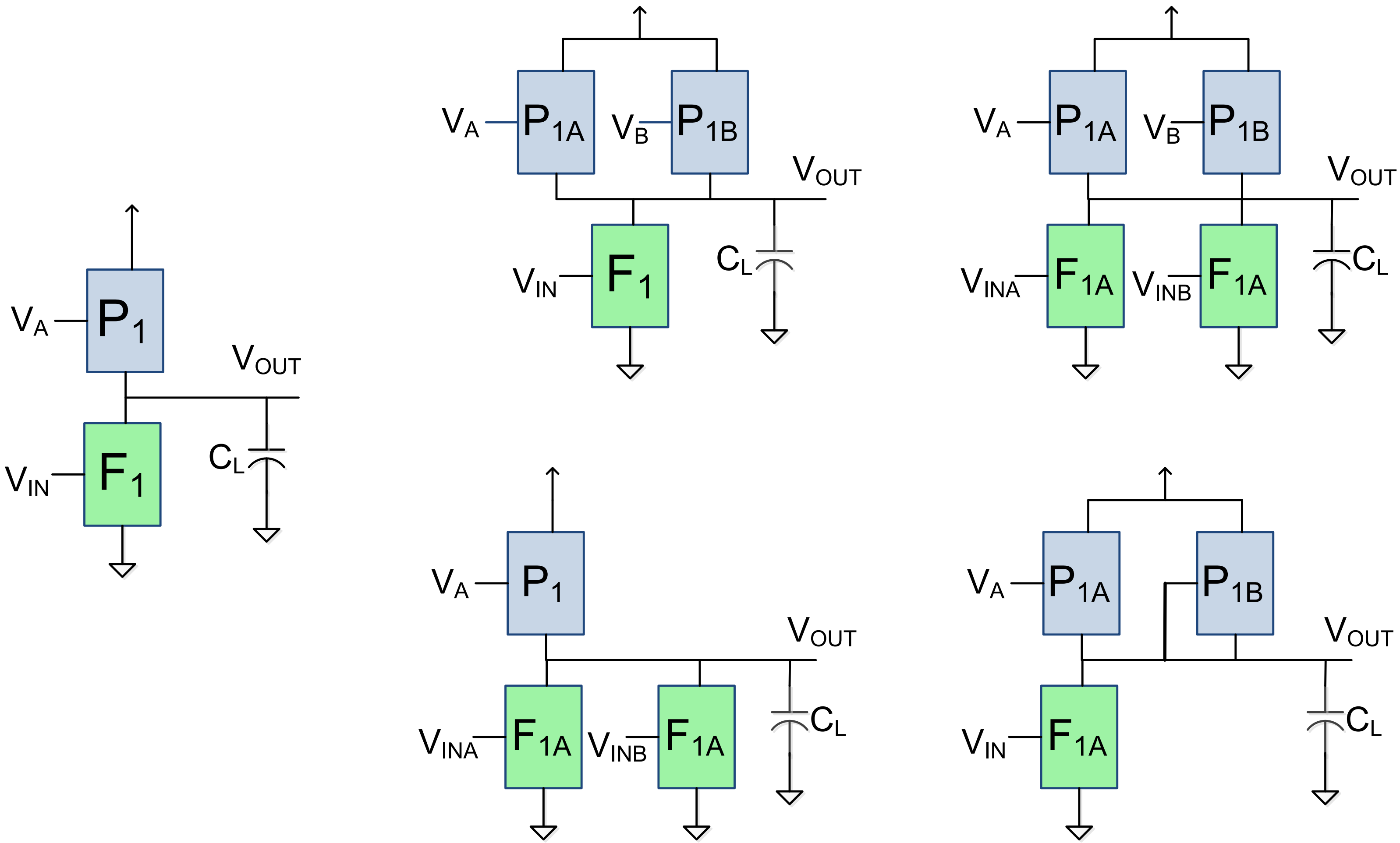
Gain Enhancement with Regenerative Feedback

The feedback performance can actually be enhanced if the open-loop amplifier is unstable **Why?**

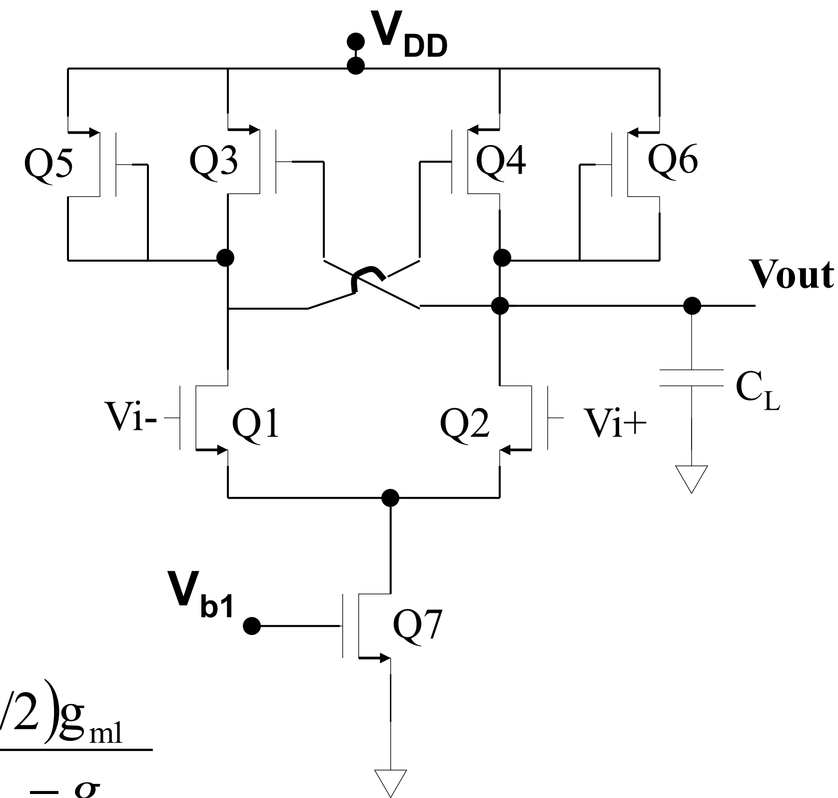


- Time required to get in settling window can be reduced with RHP pole
- But, if pole is too far in RHP, response will exit top of window

Some Half-Circuits with Interesting Potential



Existing Positive Feedback Amplifier



$$A_{VO} = \frac{(1/2)g_{m1}}{g_{o2} + g_{o4} + g_{06} + g_{m6} - g_{m4}} \approx \frac{(1/2)g_{m1}}{g_{m6} - g_{m4}}$$

$$A(s) = \frac{(1/2)g_{m1}}{sC_L + [g_{o2} + g_{o4} + g_{o6} + g_{m6} - g_{m4}]}$$

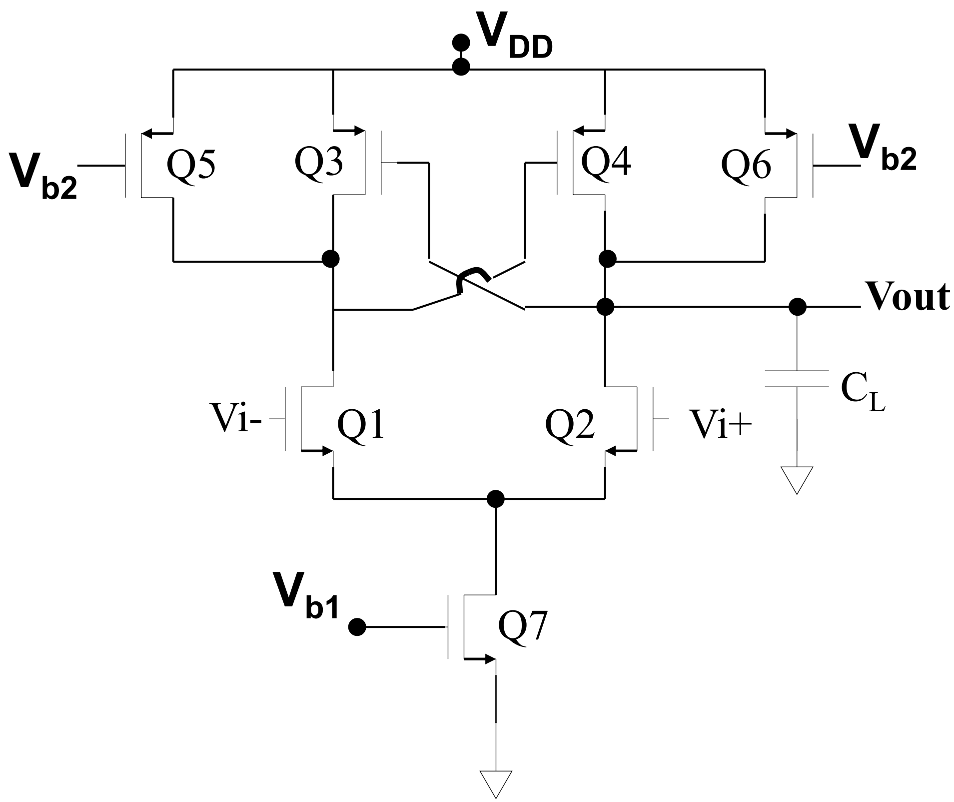
Existing Positive Feedback Amplifier

$$A_{VO} = \frac{(1/2)g_{m1}}{g_{o2} + g_{o4} + g_{o6} + g_{m6} - g_{m4}} \approx \frac{(1/2)g_{m1}}{g_{m6} - g_{m4}}$$

$$A(s) = \frac{(1/2)g_{m1}}{sC_L + [g_{o2} + g_{o4} + g_{o6} + g_{m6} - g_{m4}]}$$

- Requires precise matching of g_{m4} to $(g_{o2}+g_{o4}+g_{o6}+g_{m6})$ for good gain enhancement
- Difficult to match g_m terms to g_o-type terms

Alternate Positive Feedback Amplifier



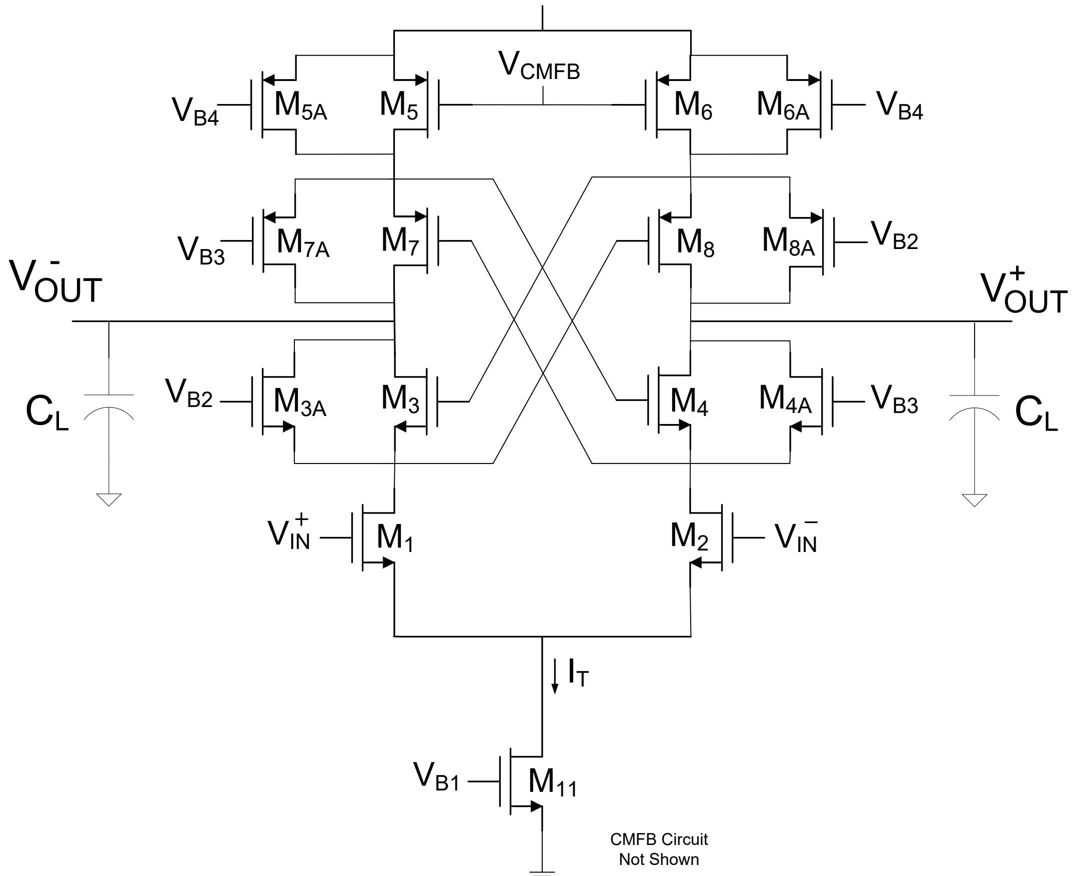
Alternate Positive Feedback Amplifier

$$A_{VO} = \frac{(1/2)g_{m1}}{g_{o2} + g_{o4} + g_{o6} - g_{m4}}$$

$$A(s) = \frac{(1/2)g_{m1}}{sC_L + [g_{o2} + g_{o4} + g_{o6} - g_{m4}]}$$

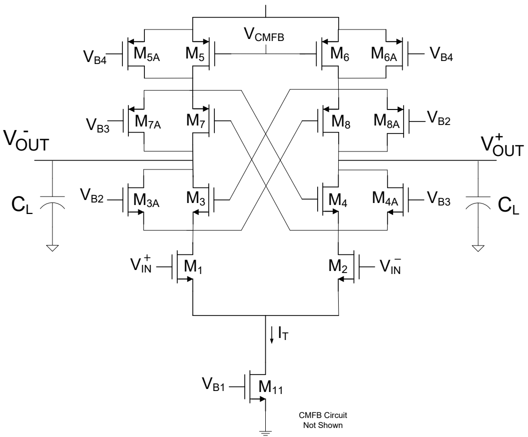
- Requires precise matching of g_{m4} to $(g_{o2}+g_{o4}+g_{o6})$ for good gain enhancement
- Difficult to match g_m terms to g_o-type terms

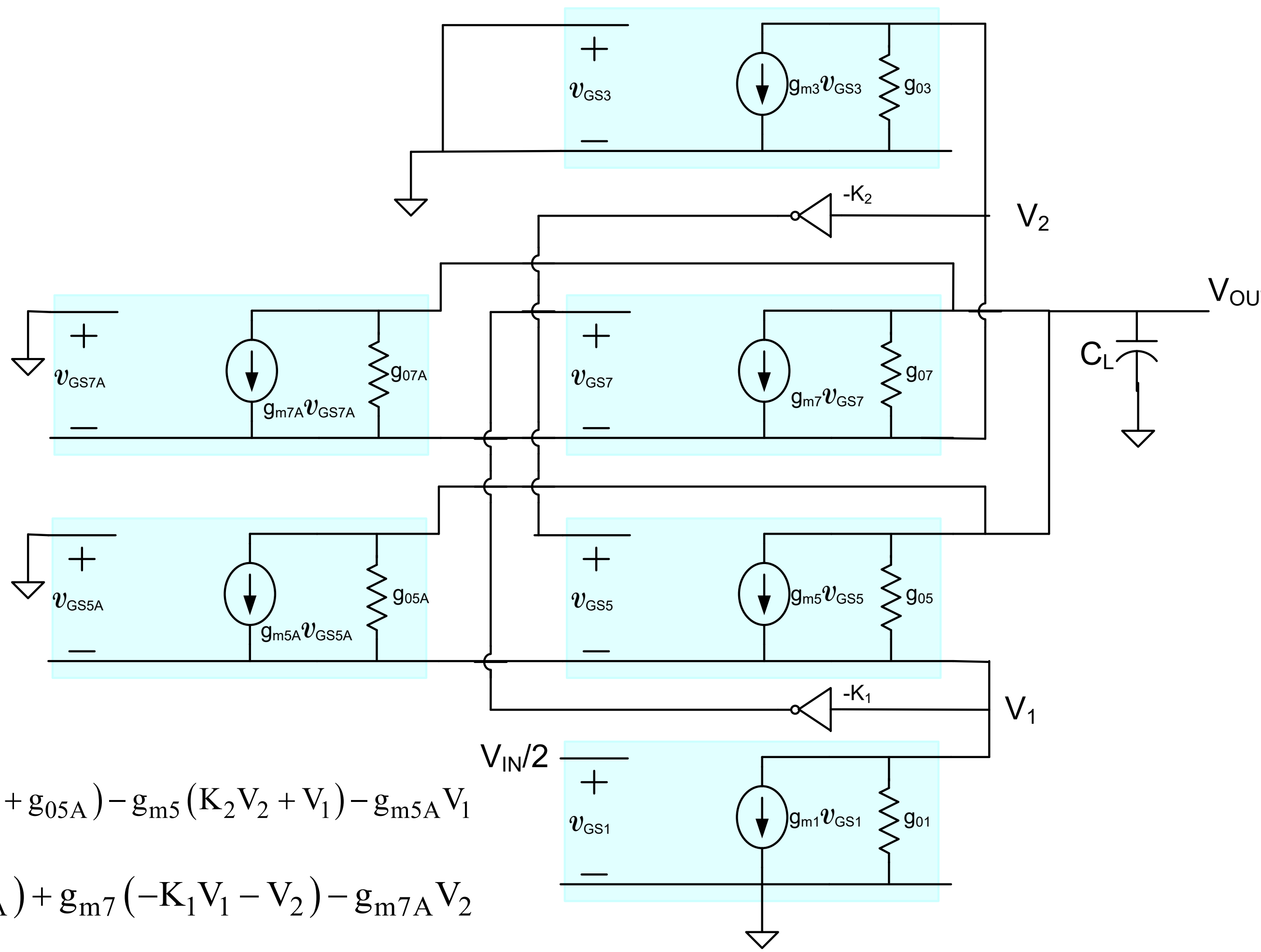
Another Positive Feedback Amplifier



- Regenerative feedback can be to either quarter circuit or counterpart circuit
- Regenerative feedback to cascode devices can significantly reduce the magnitude of the negative conductance term

Another Positive Feedback Amplifier





Small-signal half circuit

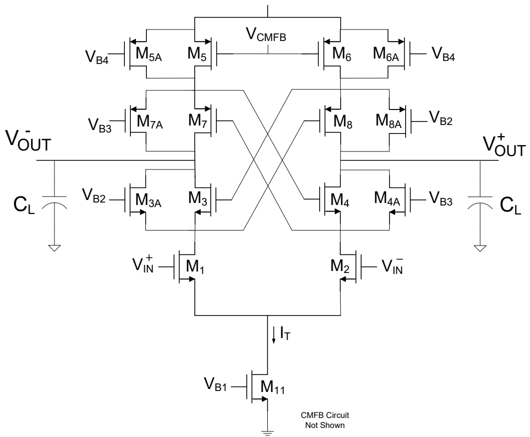
$$V_1(g_{01}+g_{05}+g_{05A})+g_{m1}V_{IN}/2=V_0(g_{05}+g_{05A})-g_{m5}(K_2V_2+V_1)-g_{m5A}V_1$$

$$V_2(g_{03}+g_{07}+g_{07A})=V_0(g_{07}+g_{07A})+g_{m7}(-K_1V_1-V_2)-g_{m7A}V_2$$

$$V_0(sC_L+g_{05}+g_{05A}+g_{07}+g_{07A})=V_2(g_{07}+g_{07A})+V_1(g_{05}+g_{05A})+g_{m7}(K_1V_1+V_2)+g_{m7A}V_2+g_{m5}(K_2V_2+V_1)+g_{m5A}V_1$$

Ki=0 if cross-coupling absent, 1 if cross-coupling present

Another Positive Feedback Amplifier



$$V_1\left(g_{01}+g_{05}+g_{05A}\right)+g_{m1}V_{IN}/2=V_0\left(g_{05}+g_{05A}\right)-g_{m5}\left(K_2V_2+V_1\right)-g_{m5A}V_1$$

$$V_2\left(g_{03}+g_{07}+g_{07A}\right)=V_0\left(g_{07}+g_{07A}\right)+g_{m7}\left(-K_1V_1-V_2\right)-g_{m7A}V_2$$

$$V_0\left(sC_L+g_{05}+g_{05A}+g_{07}+g_{07A}\right)=V_2\left(g_{07}+g_{07A}\right)+V_1\left(g_{05}+g_{05A}\right)+g_{m7}\left(K_1V_1+V_2\right)$$
$$+g_{m7A}V_2+g_{m5}\left(K_2V_2+V_1\right)+g_{m5A}V_1$$

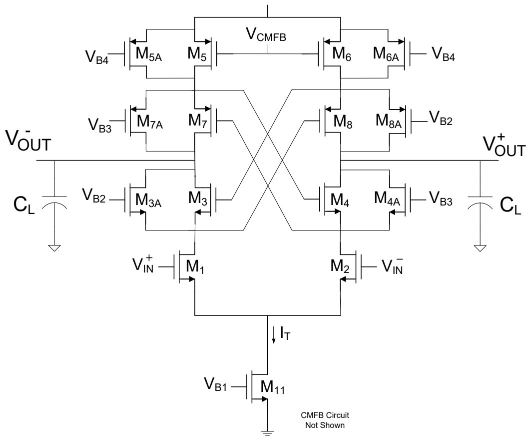
Transfer function solution with MAPLE T(s)=N(s)/S(s)

num := -(-K1 K2 gm5 gm7 + gm5 gm7 + gm7A gm5 + go7 gm5 + go3 gm5
+ go7A gm5 + K1 go3 gm7 + gm5A gm7 + go5 gm7 + go5A gm7
+ go5A go7A + go5 go7A + go5A gm7A + go5 gm7A + go5 go7
+ go5 go3 + gm5A gm7A + go5A go7 + gm5A go3 + go5A go3
+ gm5A go7A + gm5A go7) gm1

den := -go1 go7 gm5 K2 - gm7 K1 go5A go3 - gm7 K1 go5 go3
- go1 go7A gm5 K2 + (gm5A gm7A + gm7A gm5 + go5A go3
+ go5A gm7A + gm5 gm7 + go1 gm7 + go5 gm7 + go5A gm7
+ gm5A gm7 - K1 K2 gm5 gm7 + go1 go7 + gm5A go3 + go5 gm7A
+ go3 gm5 + go5 go3 + go5 go7A + go5 go7 + go1 go7A
+ go5A go7 + gm5A go7A + go5A go7A + go7 gm5 + gm5A go7
+ go7A gm5 + go1 go3 + go1 gm7A) sCL + gm7 go5 go1
+ go5A go1 go3 + gm7 go5A go1 + gm5A go7A go3 + gm5A go7 go3
+ go5A go7 go3 + go5 go7 go3 + gm5 go7 go3 + go1 go7A go3
+ go1 go7 go3 + go5A go1 go7A + go5A go1 go7 + go5 go1 go3
+ go5A go1 gm7A + go5 go1 go7 + go5 go1 gm7A + go5 go7A go3
+ go5 go1 go7A + gm5 go7A go3 + go5A go7A go3

Ki=0 if cross-coupling absent, 1 if cross-coupling present

Another Positive Feedback Amplifier



$$V_1\left(g_{01}+g_{05}+g_{05A}\right)+g_{m1}V_{IN}/2=V_0\left(g_{05}+g_{05A}\right)-g_{m5}\left(K_2V_2+V_1\right)-g_{m5A}V_1$$

$$V_2\left(g_{03}+g_{07}+g_{07A}\right)=V_0\left(g_{07}+g_{07A}\right)+g_{m7}\left(-K_1V_1-V_2\right)-g_{m7A}V_2$$

$$V_0\left(sC_L+g_{05}+g_{05A}+g_{07}+g_{07A}\right)=V_2\left(g_{07}+g_{07A}\right)+V_1\left(g_{05}+g_{05A}\right)+g_{m7}\left(K_1V_1+V_2\right)$$
$$+g_{m7A}V_2+g_{m5}\left(K_2V_2+V_1\right)+g_{m5A}V_1$$

T(s)=N(s)/D(s)

Neglecting go terms compared to gm terms, simplifies to:

num := (gm5h gm7h -K1 K2 gm5 gm7+K1 go3 gm7 +gm5h go3 + go7h gm5h
+ go5h gm7h) gm1

den := (K1 K2 gm5 gm7 - gm5h gm7h- go7h gm5h - go1 gm7h - go5h gm7h
- gm5h go3) sCL - go7h go1 go5h - go7h go1 go3 - go7h go5h go3
- go1 go5h gm7h - go1 go5h go3 - go7h gm5h go3
+ go5h gm7 K1 go3 + go7h go1 gm5 K2

Practical Comments about Positive Feedback Gain Enhancement

- Significant gain enhancement is possible but most designers avoid regenerative feedback because of unfounded concerns about closed-loop stability

- Accuracy and settling time can be improved with some regenerative feedback

- Will become more critical in emerging processes where g_m/g_o ratios degrade and where supply voltages shrink thus limiting the longstanding cascode process

- Regenerative structures can have high sensitivities

- Signal swing quite limited in some of the most basic regenerative feedback structures

- Most useful in two-stage architecture where regenerative feedback is used in first stage (effects of signal swing are reduced by gain of second stage)

Summary of Methods of Gain Enhancement

Increasing the output impedance of the amplifier

cascode, folded cascode, regulated cascode, positive feedback

Increasing the transconductance

(current mirror op amp) but it didn't really help because the output conductance increased proportionally

Driving the counterpart circuit does offer some improvements in gain

Cascading gives a multiplicative gain effect

(thousands of architectures but compensation is essential)

usually limited to a two-level cascade because of too much phase accumulation

One or more of these effects can be combined

Operational Amplifier Architectures

Most of the popular operational amplifier architectures have been introduced

Large number of different architectural choices exist with substantially different performance potential

Choice of architecture is important but judicious use of DOF is essential to obtain good performance

Few architectures offer a GB power efficiency that is better than that of the reference op amp (but some two-stage amplifiers do)

Some variants of the basic amplifier structures such as buffered output stages are commonly used in some applications

Observations about Op Amp Design

- Considerably different insight can often be obtained by viewing a circuit in multiple ways
- Various systematic procedures for designing op amps have been introduced
- It is important to understand the design space and to identify a good set of design variables
  - design spaces can be explored in many different ways but the degrees of freedom are incredibly valuable resources and should be used judiciously
- Cascaded amplifiers offer potential for gain enhancement but compensation schemes to practically work with more than two levels of cascading have not yet emerged
- Positive feedback appears to provide a promising approach for building high gain amplifiers in low voltage processes but research is ongoing into how this concept can be fully utilized

Up to this point all analysis of the op amp has focused on small-signal gain characteristics

Linearity of the amplifier does play a role in linearity and spectral performance of feedback amplifiers

Linearity is of major concern when the op amp is used open-loop such as in OTA applications

A major source of linearity is often associated with the differential input pair

Will consider linearity of the input differential pairs

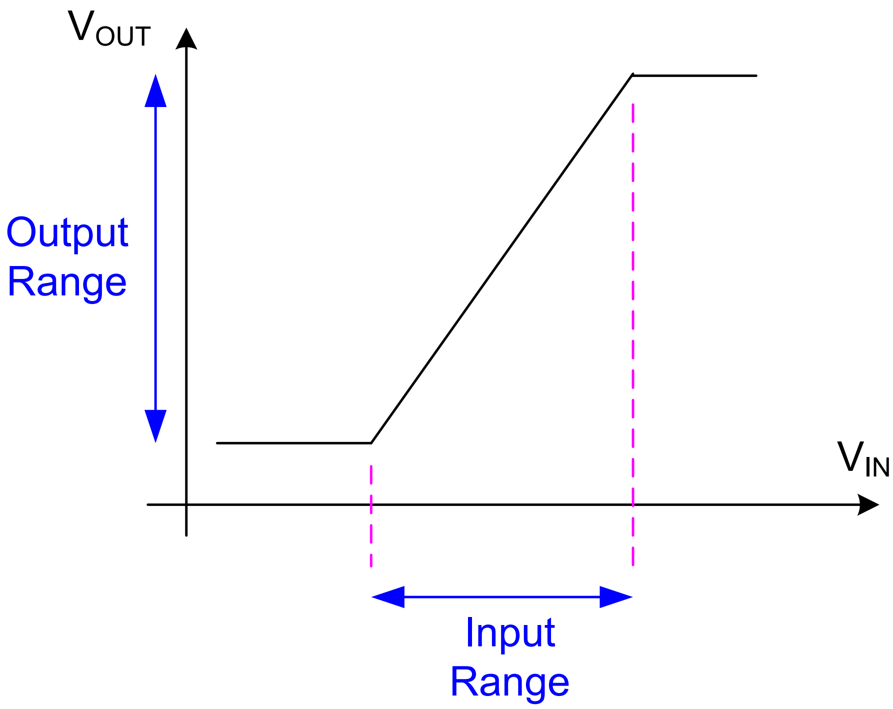
Signal Swing and Linearity

Signal swing identifies range over which signals can be applied and still maintain operation of devices in desired region of operation

Some subset of the signal swing range will be quite linear

Often that subset is close to the entire signal swing range

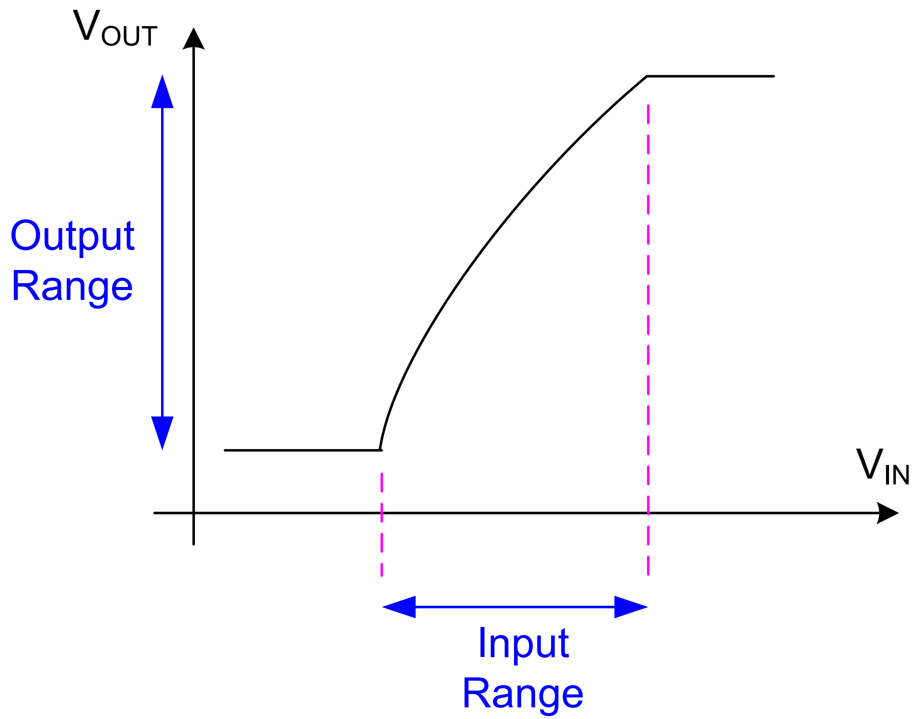
Signal Swing and Linearity



Ideal Scenario:

Completely Linear over Input and Output Range

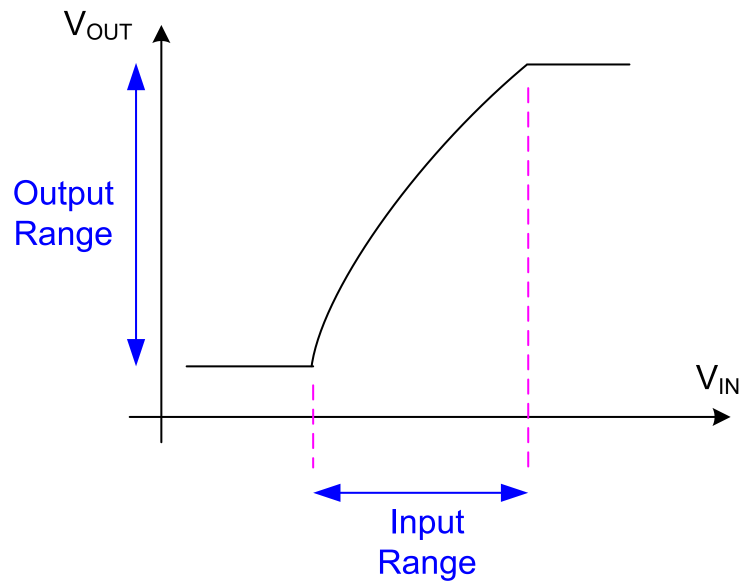
Signal Swing and Linearity

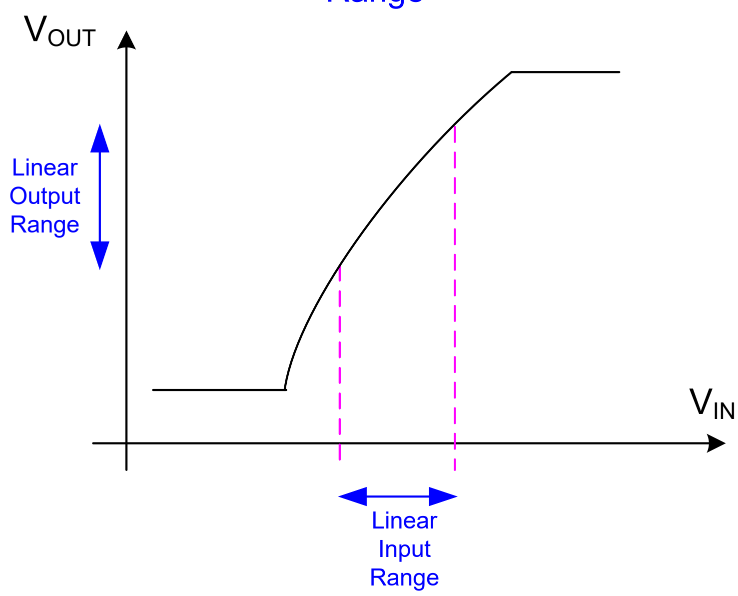


Realistic Scenario:

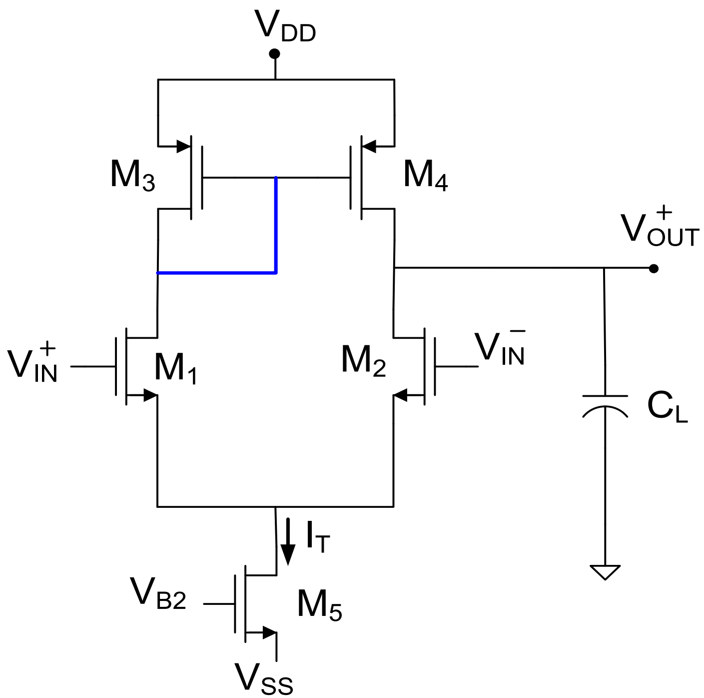
- Modest Nonlinearity throughout Input Range
- But operation will be quite linear over subset of this range

Signal Swing and Linearity

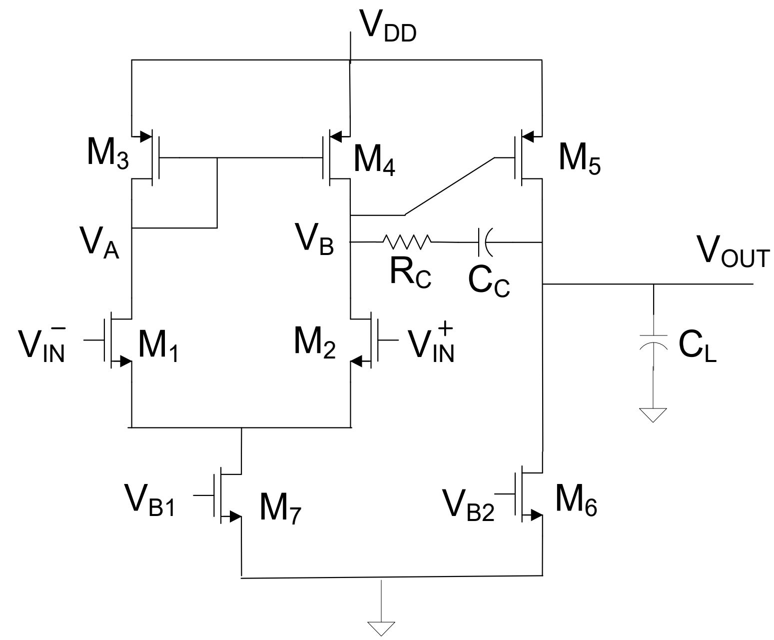




Linearity of Amplifiers



Single-Stage

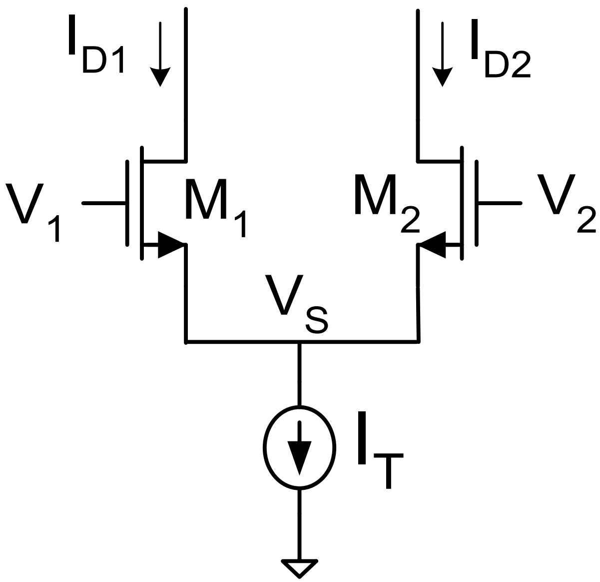


Two-Stage

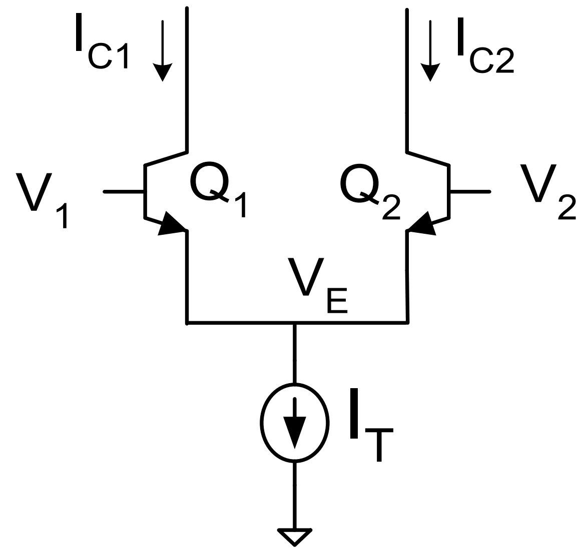
Linearity of differential pair of major concern

Linearity of common-source amplifier is of major concern (since signals so small at output of differential pair)

Differential Input Pairs

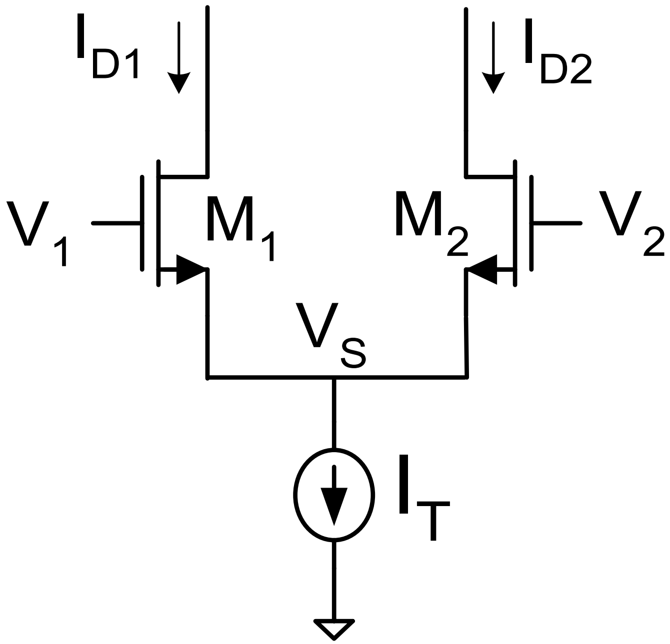


MOS Differential Pair



Bipolar Differential Pair

MOS Differential Pair



$$I_{D1} = \frac{\mu C_{OX} W}{2L}\left(V_1 - V_S - V_T\right)^2$$

$$I_{D2} = \frac{\mu C_{OX} W}{2L}\left(V_2 - V_S - V_T\right)^2$$

$$I_{D1} + I_{D2} = I_T$$

$$\pm\sqrt{I_{D1}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_1 - V_S - V_T$$

$$\pm\sqrt{I_{D2}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_2 - V_S - V_T$$

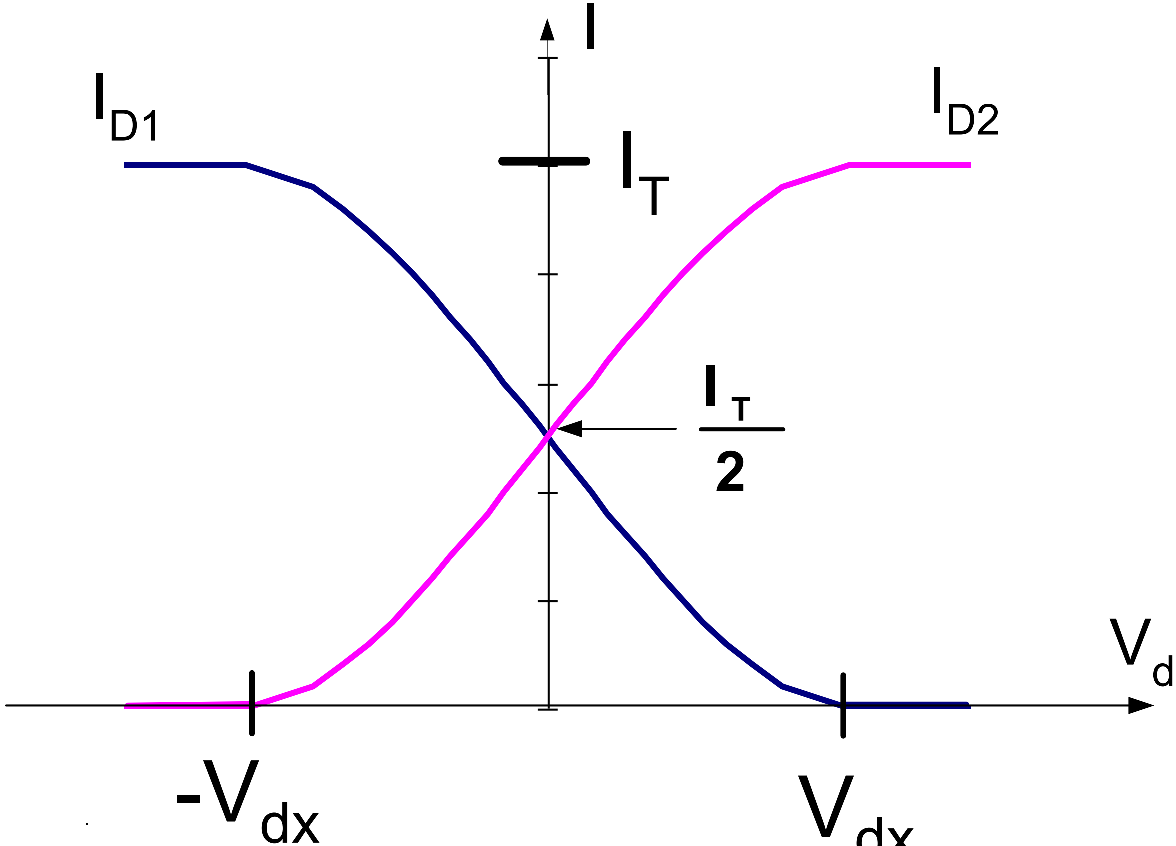
$$V_d = V_2 - V_1$$

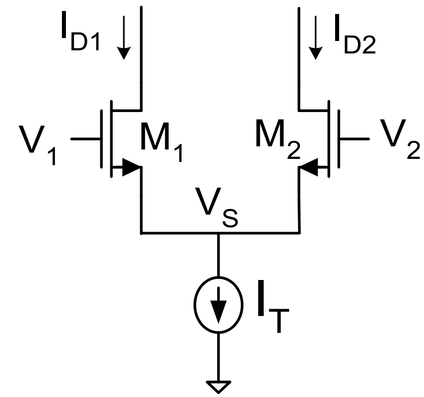
$$V_d = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$V_d = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

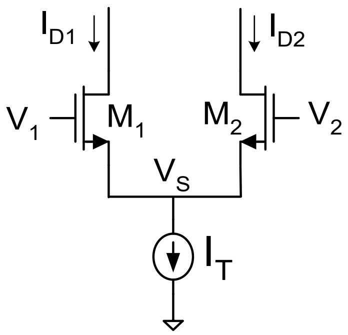
Transfer Characteristics of MOS Differential Pair

$$V_d = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$





MOS Differential Pair



$$V_d = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$V_d = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

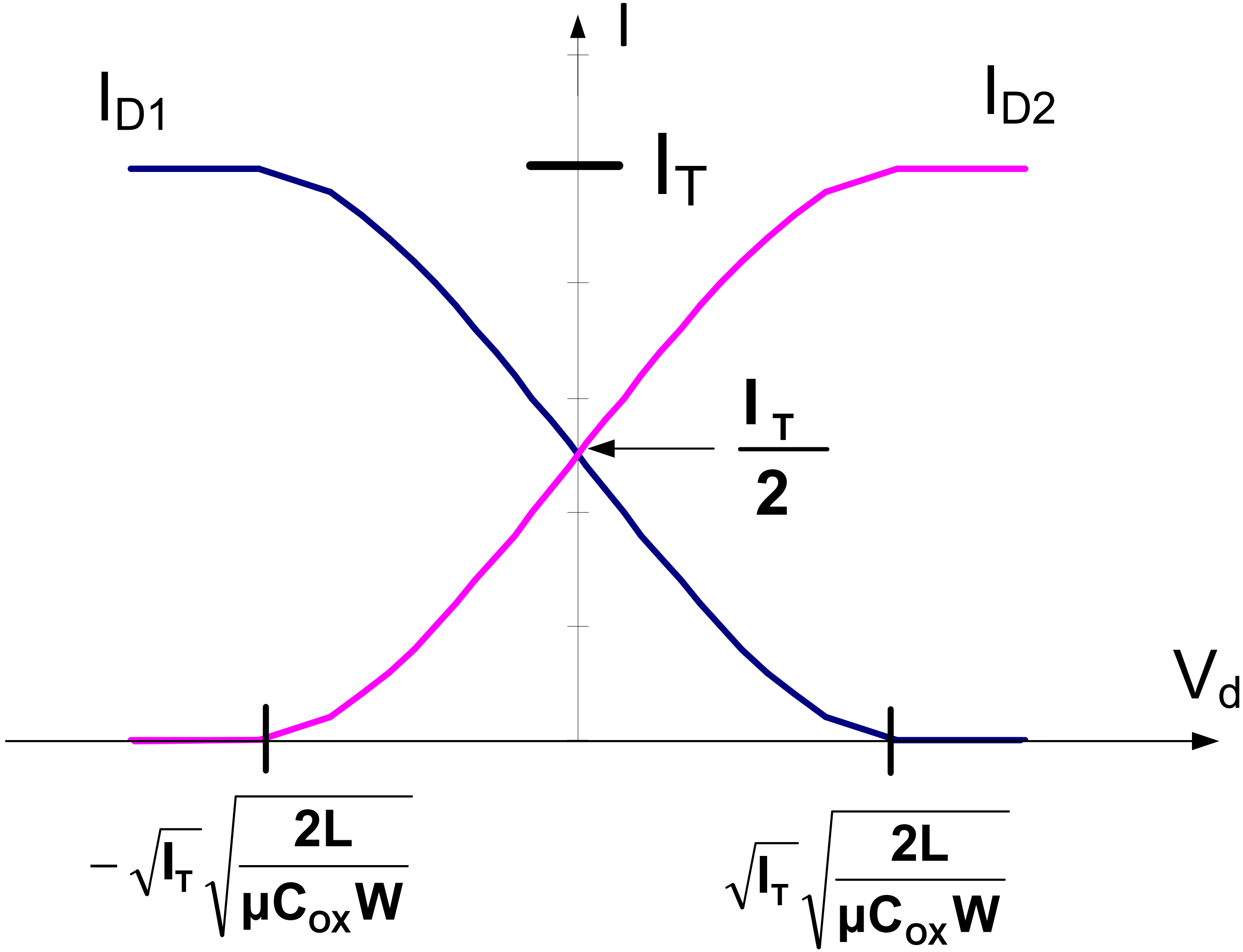
What values of V_d will cause all of the current to be steered to the left or the right ?

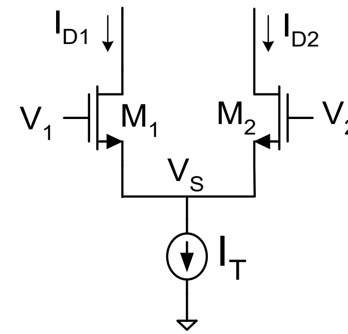
Setting $I_{D1}=0$ obtain:

$$\mathbf{V_{dx}} = \pm\sqrt{\frac{\mathbf{2L}}{\mathbf{\mu C_{OX} W}}}\left(\sqrt{\mathbf{I_T}}\right)$$

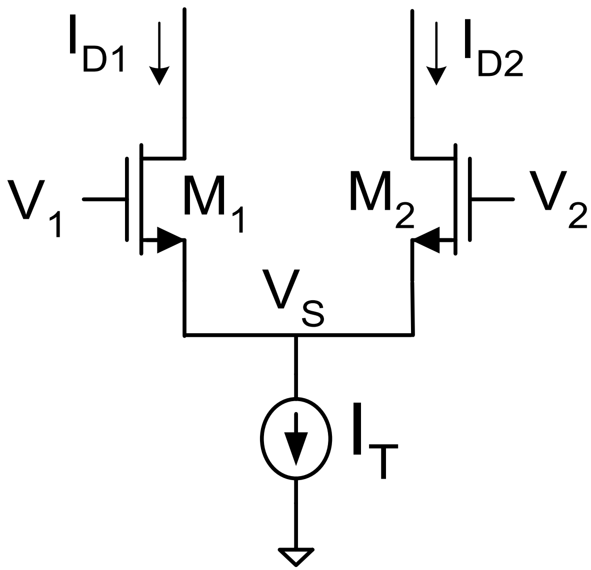
Transfer Characteristics of MOS Differential Pair

$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$





Q-point Calculations



$$V_{dx} = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T}\right)$$

- Have naturally expressed V_{dx} in natural parameter domain
- This expression does not provide good insight into actual swing

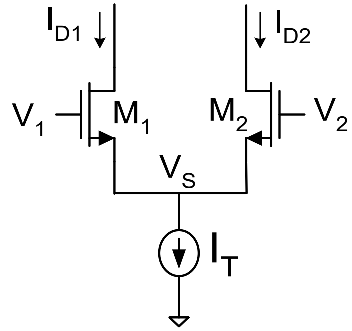
From device model:

$$\frac{I_T}{2} = \frac{\mu C_{OX} W}{2L}\left(V_{EB}\right)^2 \quad \Rightarrow \quad V_{EB} = \sqrt{I_T}\sqrt{\frac{L}{\mu C_{OX} W}}$$

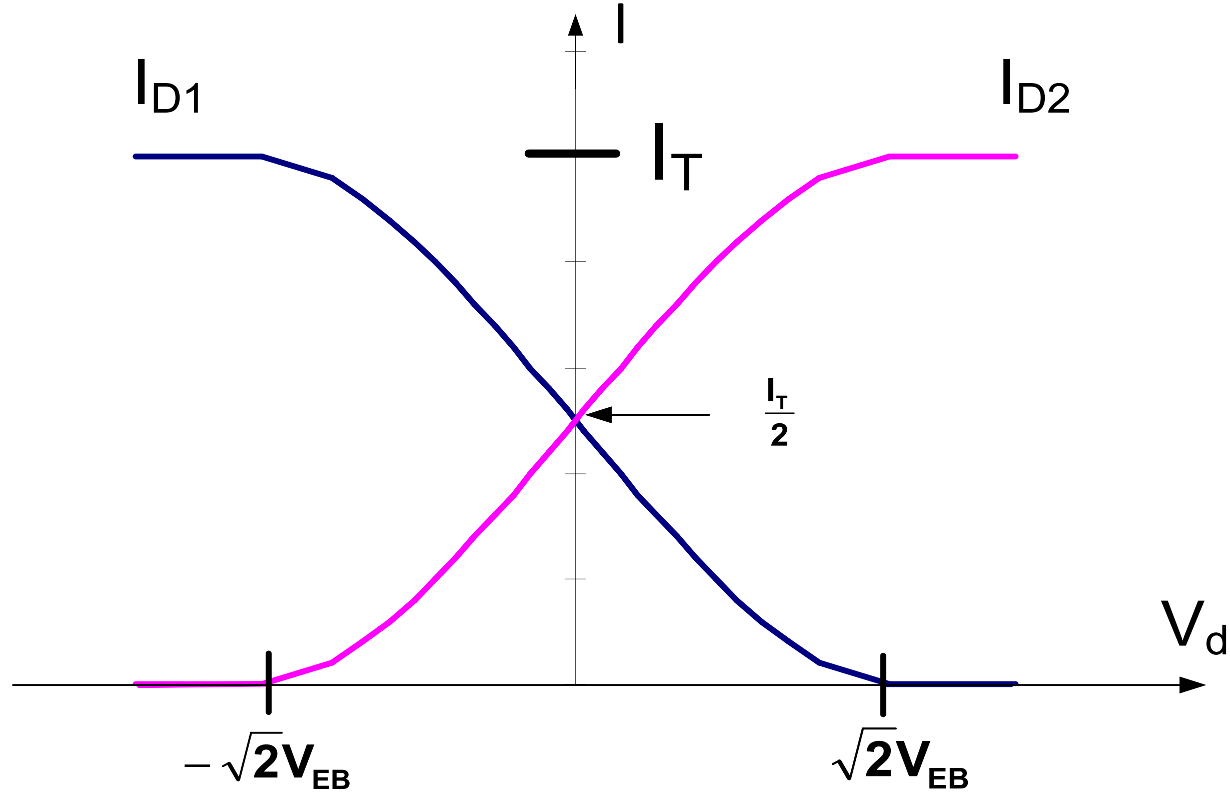
Observe !!

$$V_{dx} = \pm\sqrt{2}V_{EB}$$

Transfer Characteristics of MOS Differential Pair



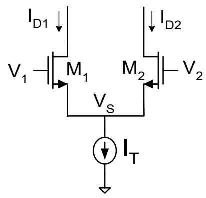
$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

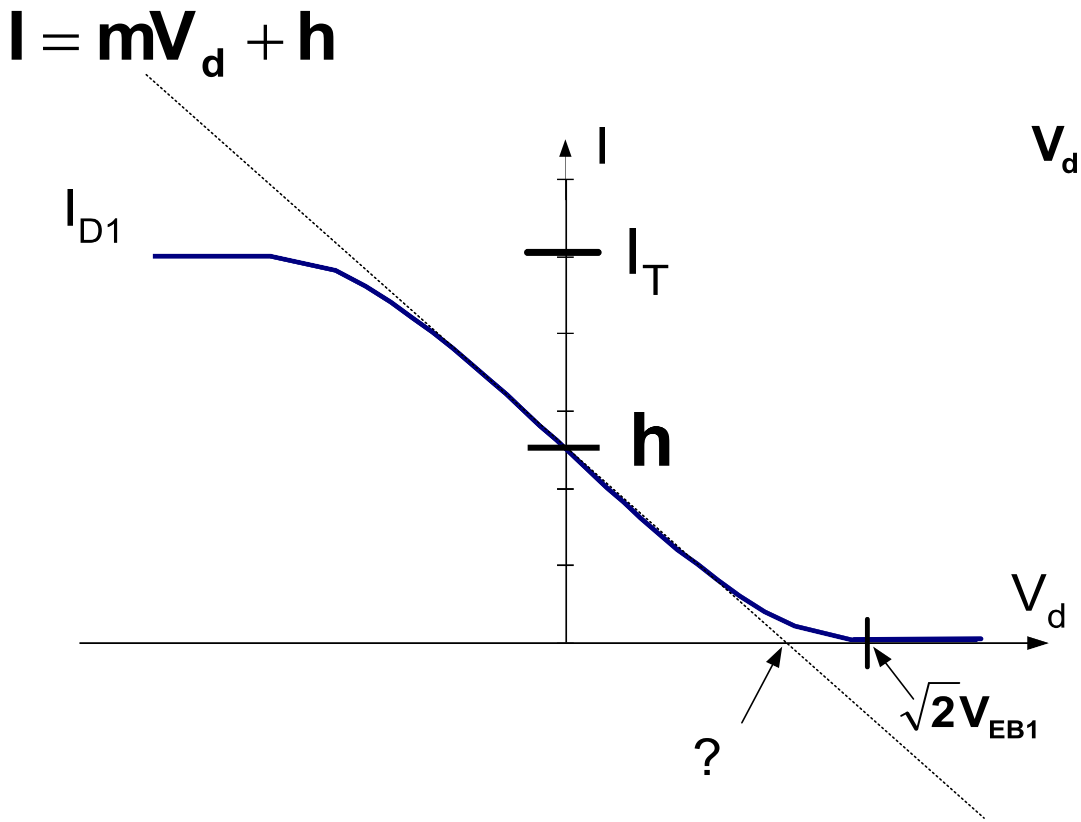


V_{EB} affects linearity

How linear is the amplifier ?

How linear is the amplifier ?





$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

Consider the fit line:

$$I = mV_d + h$$

When V_d=0, I=I_T/2, thus

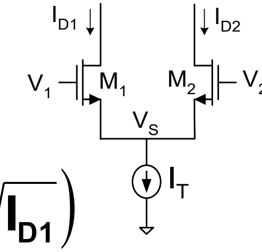
$$h = \frac{I_T}{2}$$

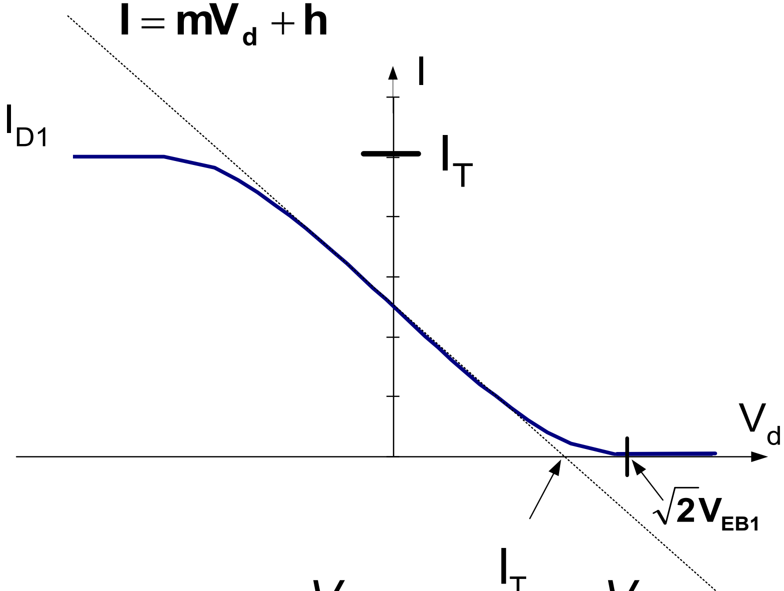
$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

Q-pt = (0,h)

How linear is the amplifier ?





$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right)$$

$$m = \left. \frac{\partial I_{D1}}{\partial V_d} \right|_{Q-pt}$$

$$\frac{\partial V_d}{\partial I_{D1}} = \sqrt{\frac{2L}{\mu C_{ox} W}} \left. \left(\frac{1}{2}(I_T - I_{D1})^{-1/2}(-1) - \frac{1}{2}(I_{D1})^{-1/2} \right) \right|_{Q-point}$$

$$\frac{\partial V_d}{\partial I_{D1}} = -2\sqrt{\frac{L}{\mu C_{ox} W}} \sqrt{\frac{1}{I_T}}$$

$$\sqrt{\frac{L}{\mu C_{ox} W}} = \frac{V_{EB1}}{\sqrt{I_T}}$$

$$\frac{\partial V_d}{\partial I_{D1}} = -2\frac{V_{EB1}}{I_T}$$

$$m = \left. \frac{\partial I_{D1}}{\partial V_d} \right|_{Q-pt} = -\frac{I_T}{2V_{EB1}}$$

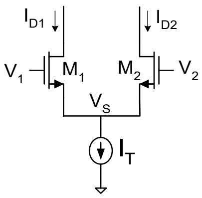
$$V_{dint} = -\frac{I_T}{2m} = V_{EB1}$$

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

Thus fit line is:

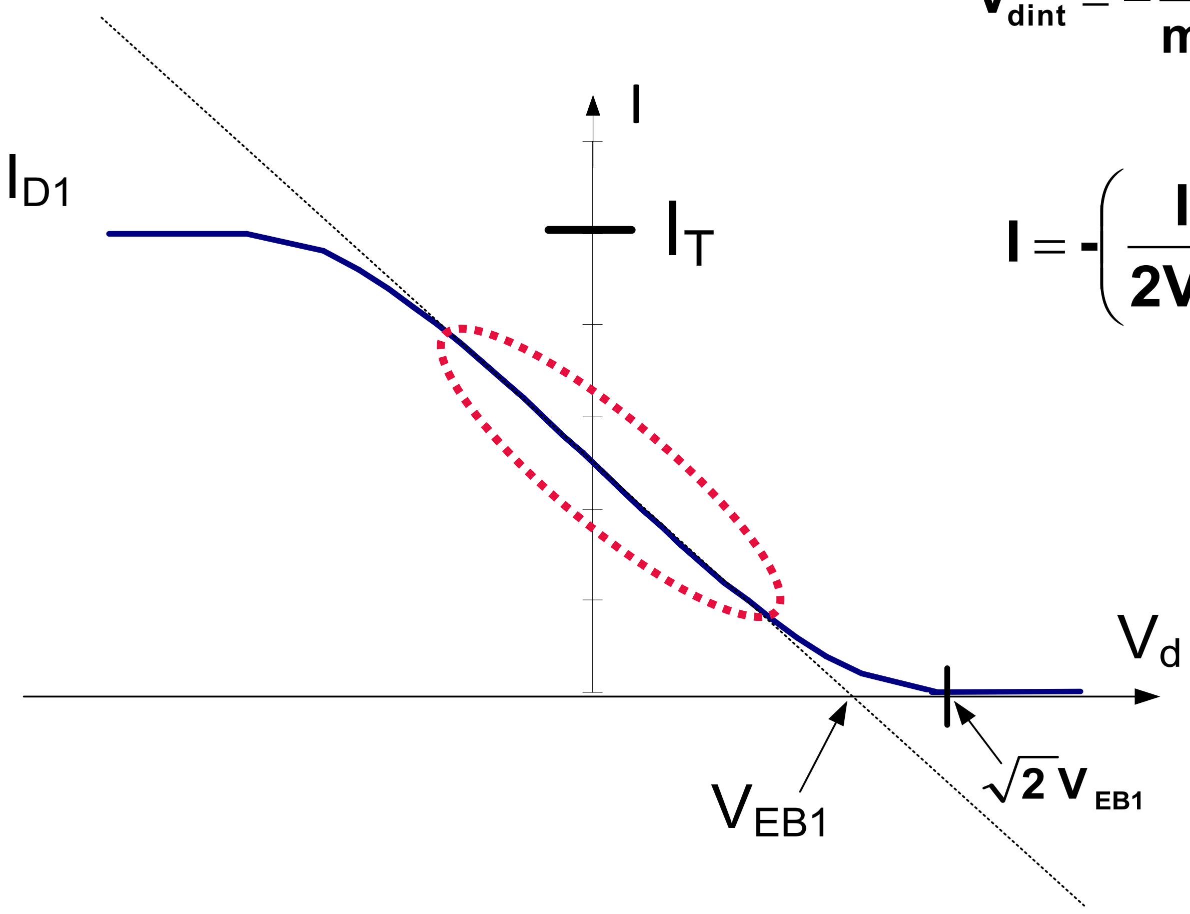
$$I = -\frac{I_T}{2V_{EB1}} V_d + \frac{I_T}{2}$$

How linear is the amplifier ?

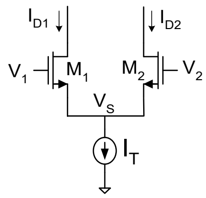


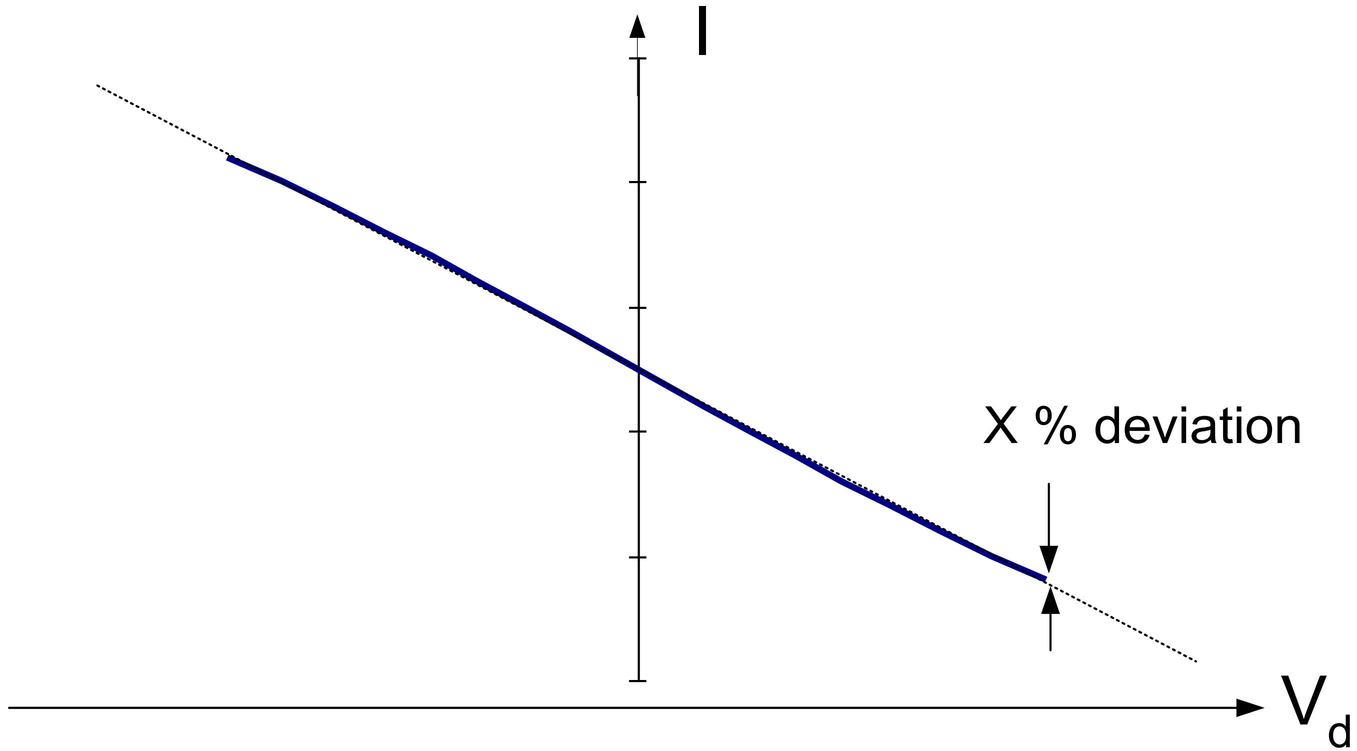
$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1}$$

$$I = -\left(\frac{I_T}{2V_{EB1}}\right)V_d + \frac{I_T}{2}$$



How linear is the amplifier ?

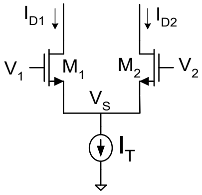


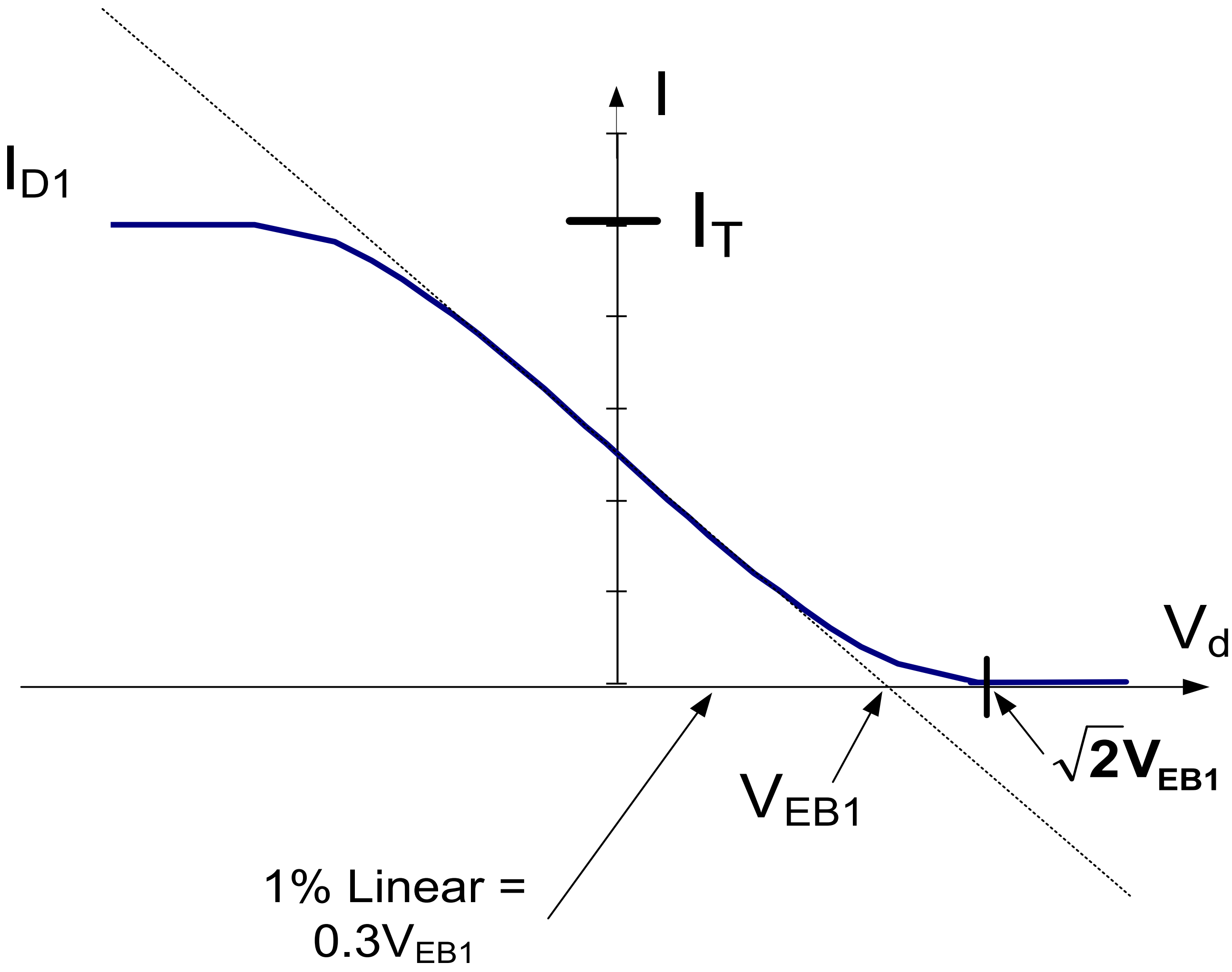


It can be shown that a 1% deviation from the straight line occurs at

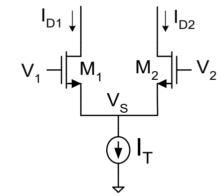
$V_d \cong \frac{V_{EB}}{3}$ and a 0.1% variation occurs at $V_d \cong \frac{V_{EB}}{10}$

How linear is the amplifier ?

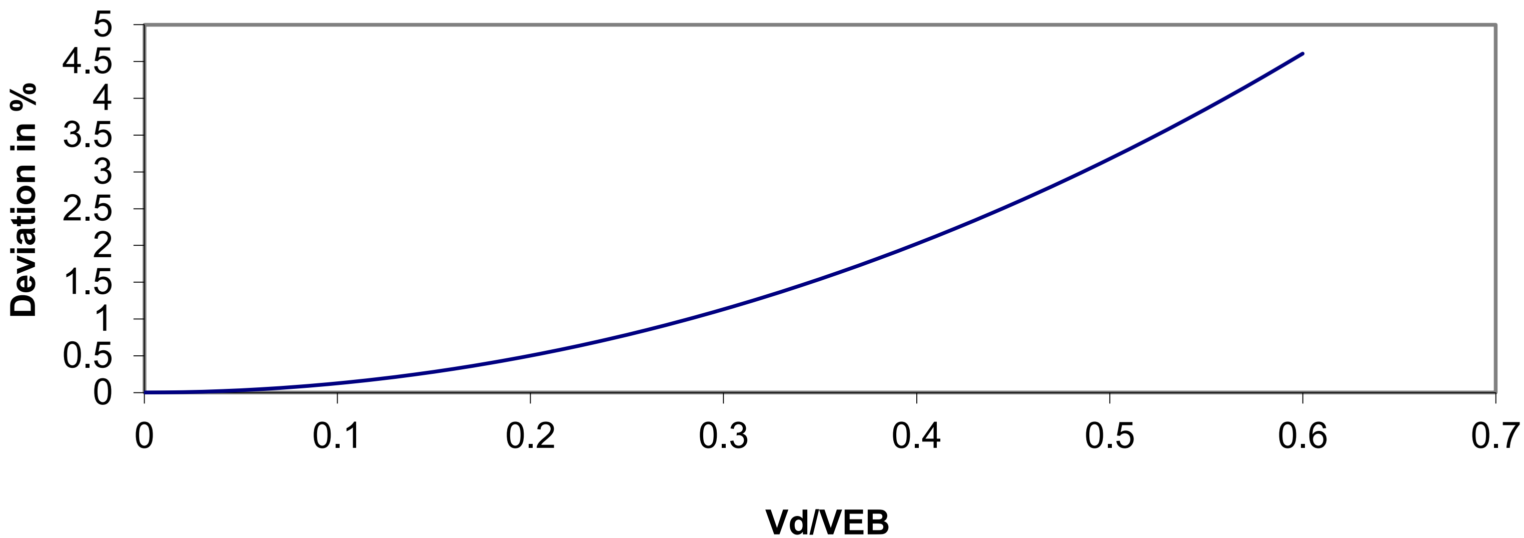




How linear is the amplifier ?



Deviation from Linear



Vd/VEB	θ	Vd/VEB	θ	Vd/VEB	θ
0.02	0.005	0.22	0.607	0.42	2.23
0.04	0.020	0.24	0.723	0.44	2.45
0.06	0.045	0.26	0.849	0.46	2.68
0.08	0.080	0.28	0.985	0.48	2.92
0.1	0.125	0.3	1.13	0.5	3.18
0.12	0.180	0.32	1.29	0.52	3.44
0.14	0.245	0.34	1.46	0.54	3.71
0.16	0.321	0.36	1.63	0.56	4.00
0.18	0.406	0.38	1.82	0.58	4.30
0.2	0.501	0.4	2.02	0.6	4.61

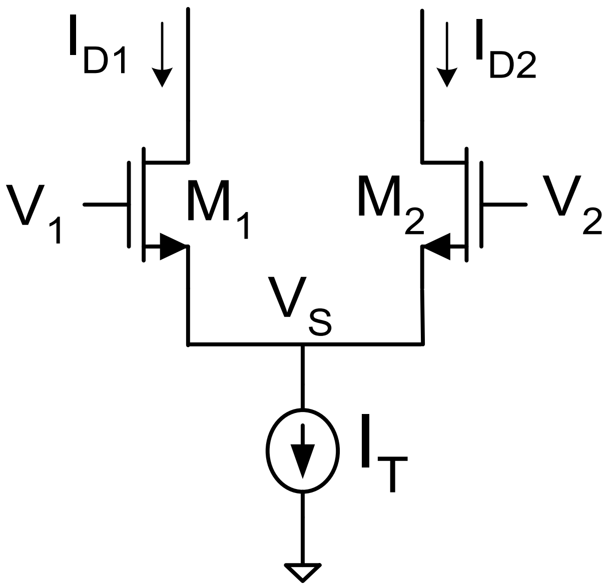
How linear is the amplifier ?

Distortion in the differential pair is another useful metric for characterizing linearity of I_{D1} and I_{D2} with sinusoidal differential excitation

Consider again the differential pair and assume excited differentially with

$$V_2 = \frac{V_d}{2} \qquad V_1 = -\frac{V_d}{2}$$

and assume $V_d = V_m \sin(\omega t)$



$$\mathbf{V_d = V_2 - V_1}$$

Recall:

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

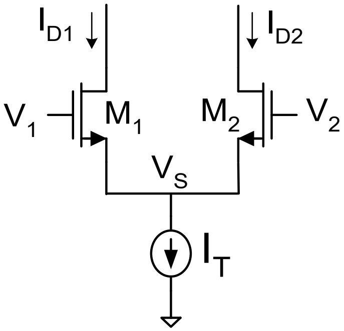
Define (strictly for notational convenience)

$$\theta = \frac{\mu C_{OX} W}{2L}$$

Thus can express as

$$\sqrt{\theta} V_d = \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}$$

How linear is the amplifier ?



$$V_d = V_m \sin(\omega t) \qquad \theta = \frac{\mu C_{OX} W}{2L}$$

$$\sqrt{\theta} V_d = \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}$$

Squaring, regrouping, and squaring we obtain

$$\theta V_d^2 = I_{D2} + (I_T - I_{D2}) - 2\sqrt{I_{D2}}\sqrt{I_T - I_{D2}}$$

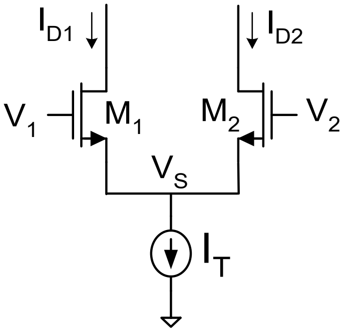
$$\theta V_d^2 = I_T - 2\sqrt{I_{D2}}\sqrt{I_T - I_{D2}}$$

$$\left(\theta V_d^2 - I_T\right)^2 = 4 I_{D2}\left(I_T - I_{D2}\right)$$

This latter equation can be expressed as a second-order polynomial in I_{D2} as

$$I_{D2}^2 - I_{D2} I_T + \left(\frac{\theta V_d^2 - I_T}{2}\right)^2 = 0$$

How linear is the amplifier ?



and assume $V_d = V_m \sin(\omega t)$ $\theta = \dfrac{\mu C_{OX} W}{2L}$

$$I_{D2}^2 - I_{D2} I_T + \left(\frac{\theta V_d^2 - I_T}{2} \right)^2 = 0$$

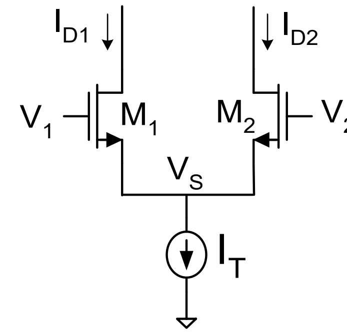
Solving, we obtain

$$I_{D2} = \frac{I_T}{2} + \sqrt{\left(\frac{I_T}{2} \right)^2 - \left(\frac{\theta V_d^2 - I_T}{2} \right)^2}$$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\left(\frac{I_T}{2} \right)^2 - \left(\frac{\theta V_d^2}{2} \right)^2 - \left(\frac{I_T}{2} \right)^2 + \frac{\theta I_T}{2} V_d^2}$$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\frac{\theta I_T}{2} V_d^2 - \left(\frac{\theta V_d^2}{2} \right)^2}$$

How linear is the amplifier ?



and assume $V_d = V_m \sin(\omega t)$ $\theta = \dfrac{\mu C_{OX} W}{2L}$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\frac{\theta I_T}{2} V_d^2 - \left(\frac{\theta V_d^2}{2}\right)^2}$$

This can be expressed as

$$I_{D2} = \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \sqrt{1 - V_d^2 \frac{\theta}{2I_T}}$$

Recall for x small

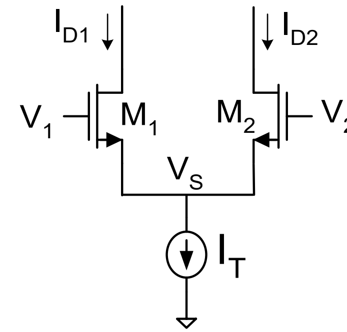
$$\sqrt{1-x} \cong 1 - \frac{x}{2} - \frac{x^2}{8} + \ldots$$

Using a Truncated Taylor's series, we obtain:

$$I_{D2} \simeq \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4I_T}\right)$$

Note this has no second-order term thus the dominant distortion when $V_d = V_m \sin(\omega t)$ will be due to the third-order term

How linear is the amplifier ?



$$I_{D2} \simeq \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4 I_T} \right)$$

$$\theta = \frac{\mu C_{OX} W}{2L}$$

Substituting in $V_d = V_m \sin(\omega t)$

$$I_{D2} \simeq \frac{I_T}{2} + V_m \sin(\omega t) \sqrt{\frac{\theta I_T}{2}} \left(1 - V_m^2 \sin^2(\omega t) \frac{\theta}{4 I_T} \right)$$

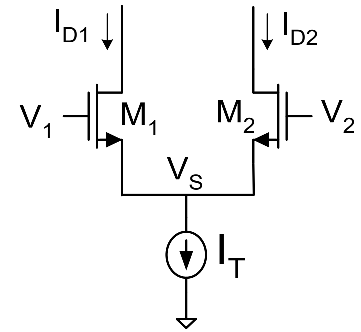
$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} \right] \sin(\omega t) - \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{4\sqrt{2}\sqrt{I_T}} \right] \sin^3(\omega t)$$

$$\sin^3(\omega t) = \frac{3}{4} \sin(\omega t) - \frac{1}{4} \sin(3\omega t)$$

$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} \right] \sin(\omega t) - \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{4\sqrt{2}\sqrt{I_T}} \right] \left[\frac{3}{4} \sin(\omega t) - \frac{1}{4} \sin(3\omega t) \right]$$

$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \sin(\omega t) + \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \left[\sin(3\omega t) \right]$$

How linear is the amplifier ?



$$I_{D2} \simeq \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4 I_T} \right) \qquad \theta = \frac{\mu C_{OX} W}{2L}$$

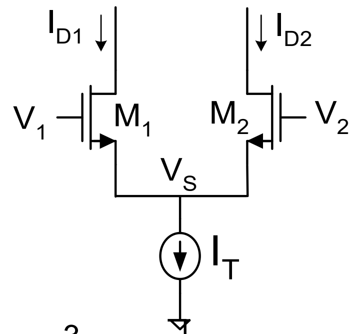
$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \sin(\omega t) + \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] [\sin(3\omega t)]$$

Note this has no second-order harmonic term thus the dominant distortion when $V_d = V_m \sin(\omega t)$ will be due to the third-order harmonic

$$I_{D2} \simeq a_0 + a_1 \sin(\omega t) + a_3 (3\omega t)$$

$$a_1 = \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \qquad a_3 = \left[\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] V_m^3$$

How linear is the amplifier ?



$$I_{D2} \simeq a_0 + a_1 \sin(\omega t) + a_3 \sin(3\omega t)$$

$$a_1 = \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \qquad a_3 = \left[\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] V_m^3$$

$$THD = 20\log\left(\frac{\sqrt{\sum_{k=2}^{\infty} a_k^2}}{a_1} \right)$$

For low distortion want THD a large negative number

Substituting in we obtain

$$THD = 20\log\left(\frac{\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} V_m^3}{V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}} \right)$$

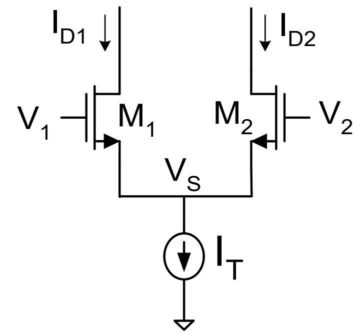
where $\theta = \dfrac{\mu C_{OX} W}{2L}$

This expression gives little insight.

Consider expression in the practical parameter domain:

$$I_T = \frac{\mu C_{OX} W}{L} V_{EB1}^2$$

How linear is the amplifier ?



$$I_{D2} \simeq a_0 + a_1 \sin(\omega t) + a_3 \sin(3\omega t)$$

$$THD = 20\log\left(\frac{\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} V_m^3}{V_m\sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}}\right)$$

$$\theta = \frac{\mu C_{OX} W}{2L}$$

$$I_T = \frac{\mu C_{OX} W}{L} V_{EB1}^2$$

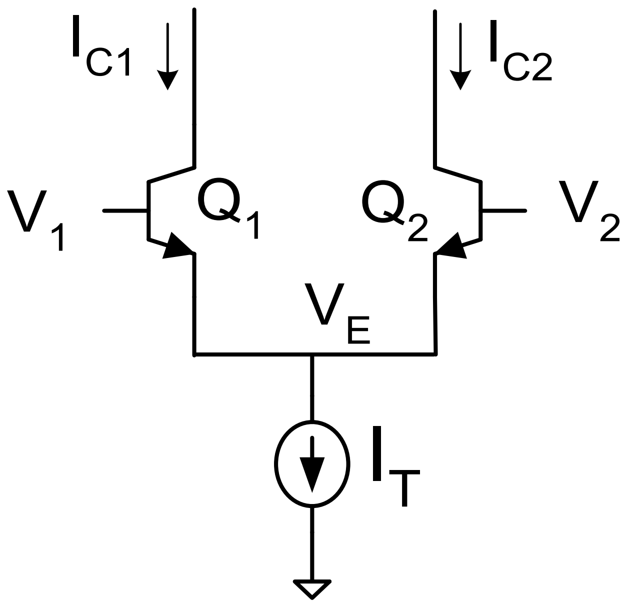
Eliminating I_T and θ, we obtain

$$THD = -20\log\left(32\left(\frac{V_{EB1}}{V_m}\right)^2 - 3\right)$$

V_m/V_{EB1}	THD (dB)
2.5	-6.52672
1	-29.248
0.5	-41.9382
0.25	-54.1344
0.1	-70.0949
0.05	-82.1422
0.025	-94.1849
0.01	-110.103

Thus to minimize THD, want V_{EB} large and V_m small

Bipolar Differential Pair



$$\left.\begin{aligned} I_{C1} &= J_S A_{E1} e^{\frac{V_1 - V_E}{V_t}} \\ I_{C2} &= J_S A_{E2} e^{\frac{V_2 - V_E}{V_t}} \\ I_{C1} + I_{C2} &= I_T \end{aligned}\right\}$$

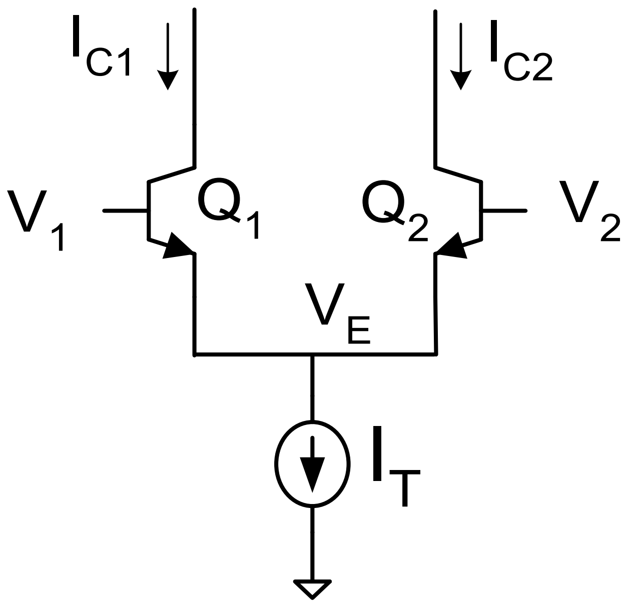
$$V_1 = V_E + V_t \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)$$

$$V_2 = V_E + V_t \ln\left(\frac{I_{C2}}{J_S A_{E2}}\right)$$

$$V_d = V_2 - V_1$$

$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

Bipolar Differential Pair



$$V_d = V_2 - V_1$$

$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

$$V_d = V_t \ln\left(\frac{I_{C2}}{I_T - I_{C2}}\right)$$

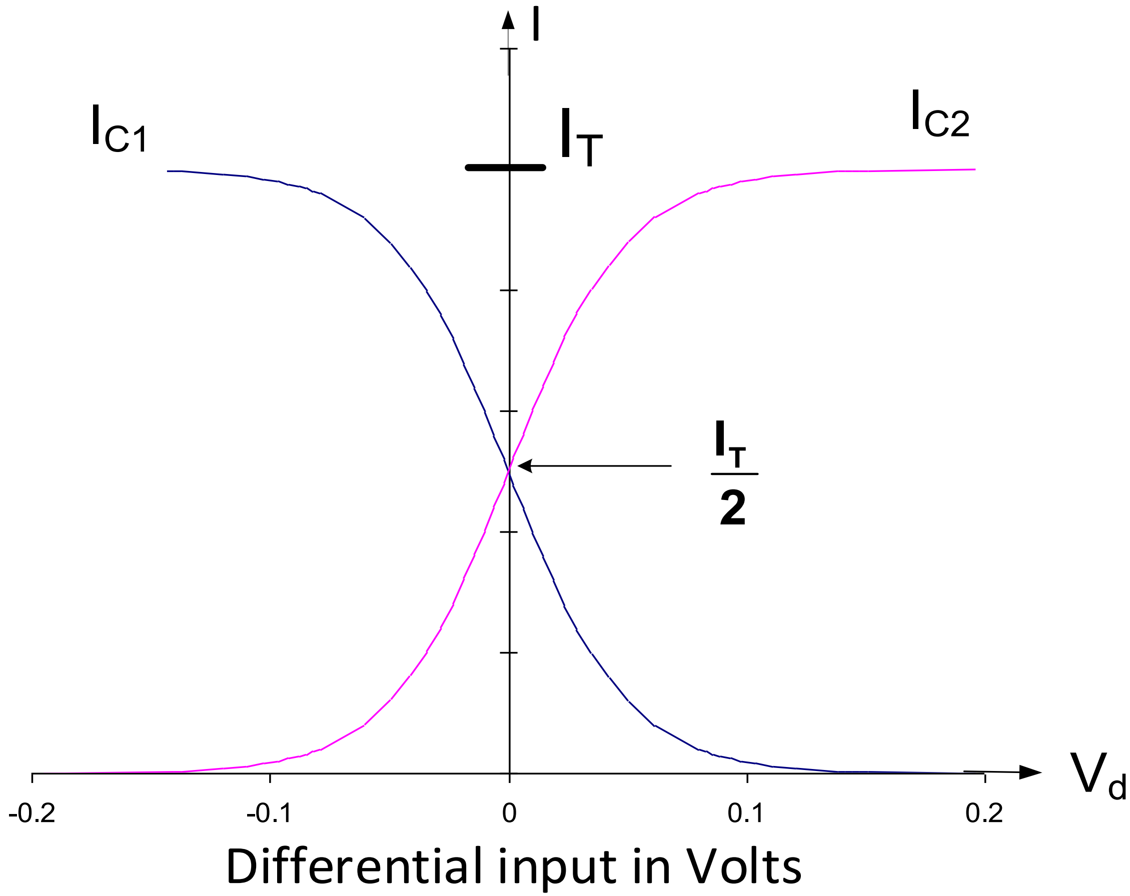
At $I_{C1}=I_{C2}=I_T/2$, $V_d=0$

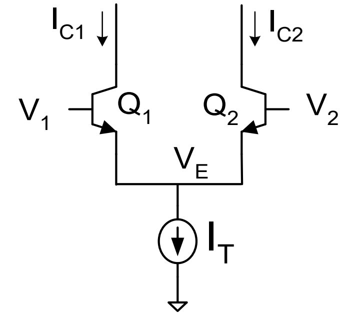
As I_{C1} approaches 0, V_d approaches infinity

As I_{C1} approaches I_T, V_d approaches minus infinity

Transition much steeper than for MOS case

Transfer Characteristics of Bipolar Differential Pair





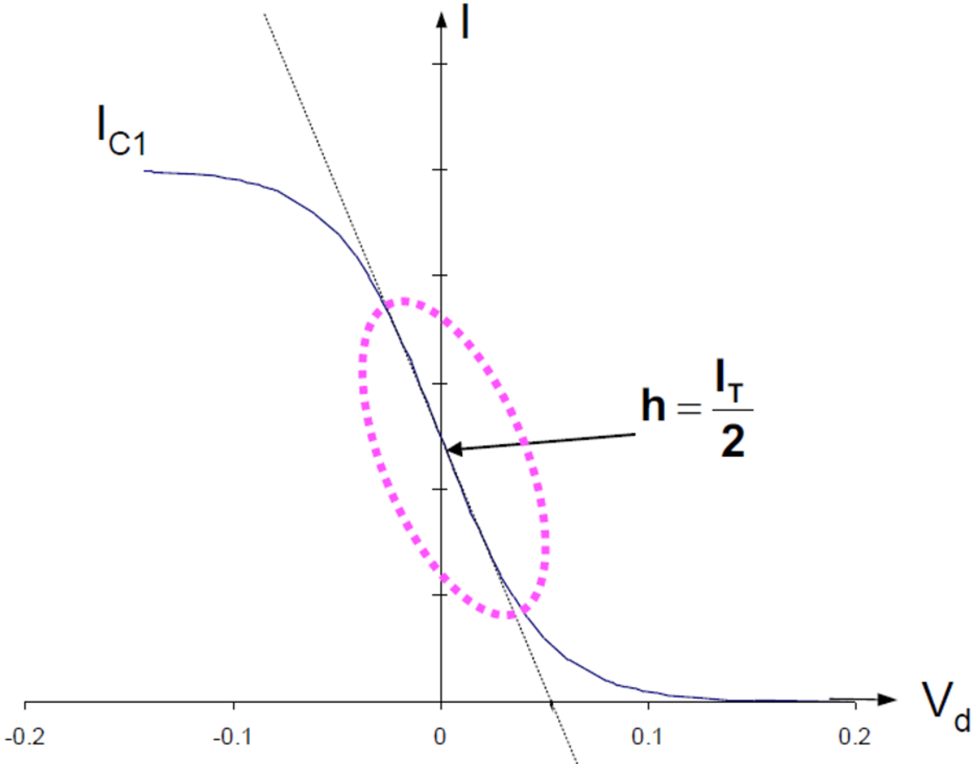
$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

Transition much steeper than for MOS case

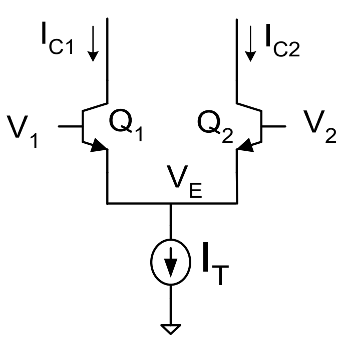
Asymptotic Convergence to 0 and I_T

Signal Swing and Linearity of Bipolar Differential Pair

$$I_{FIT} = mV_d + h$$



$$V_{d\,int} = -\frac{h}{m} = ?$$



$$\mathbf{V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)}$$

$$m = \left.\frac{\partial I_{C1}}{\partial V_d}\right|_{Q-point}$$

Q-pt = (0,h)

$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q=point} = -V_t \left.\frac{I_T}{I_{C1}\left(I_T - I_{C1}\right)}\right|_{I_{C1}=\frac{I_T}{2}}$$

$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q=point} = -\frac{4V_t}{I_T}$$

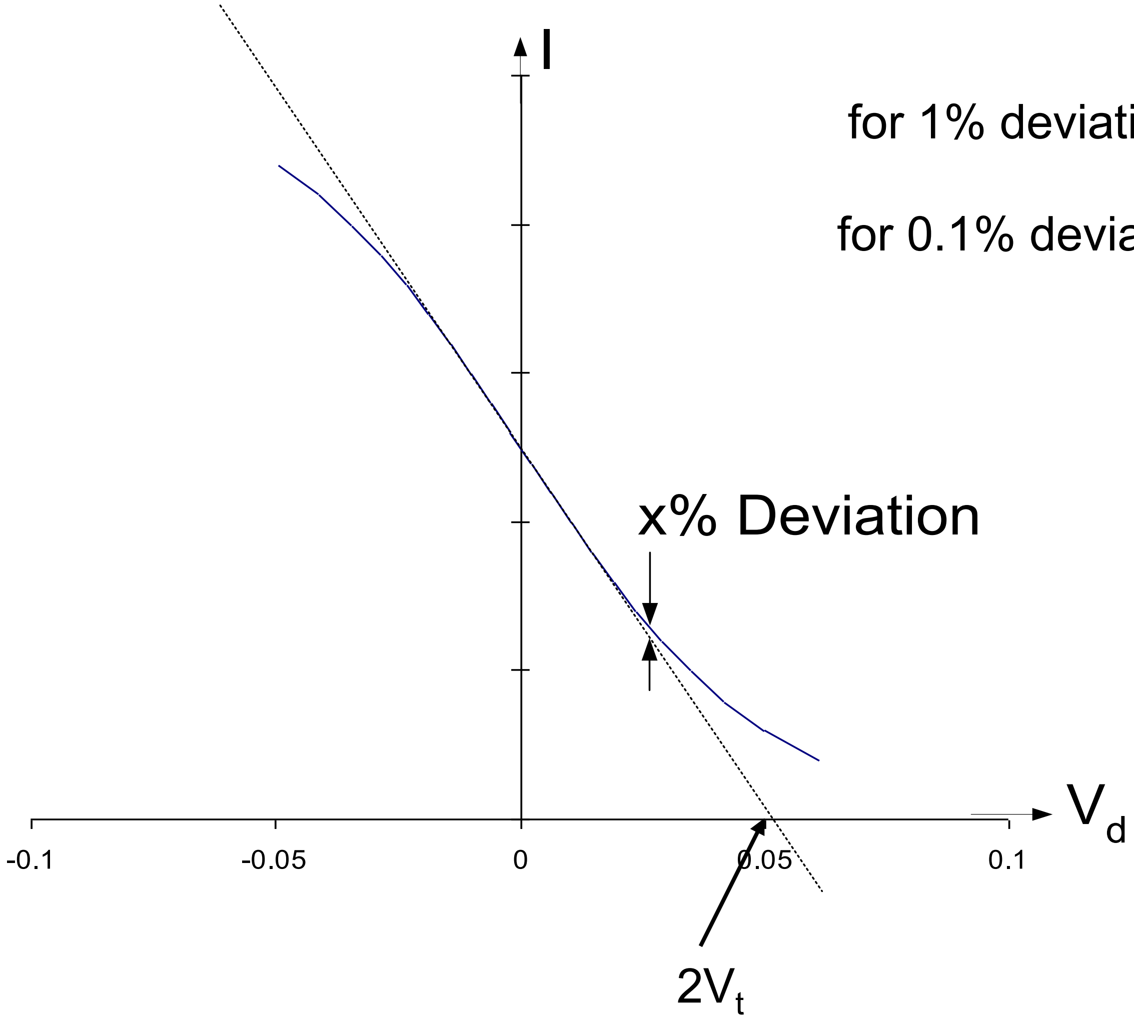
$$I_{FIT} = -\frac{I_T}{4V_t}V_d + \frac{I_T}{2}$$

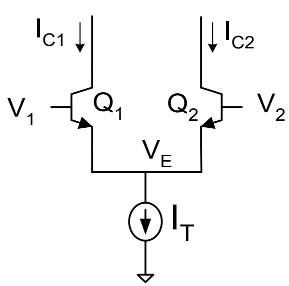
$$V_{d\,int} = -\frac{h}{m} = 2V_t$$

Signal Swing and Linearity of Bipolar Differential Pair

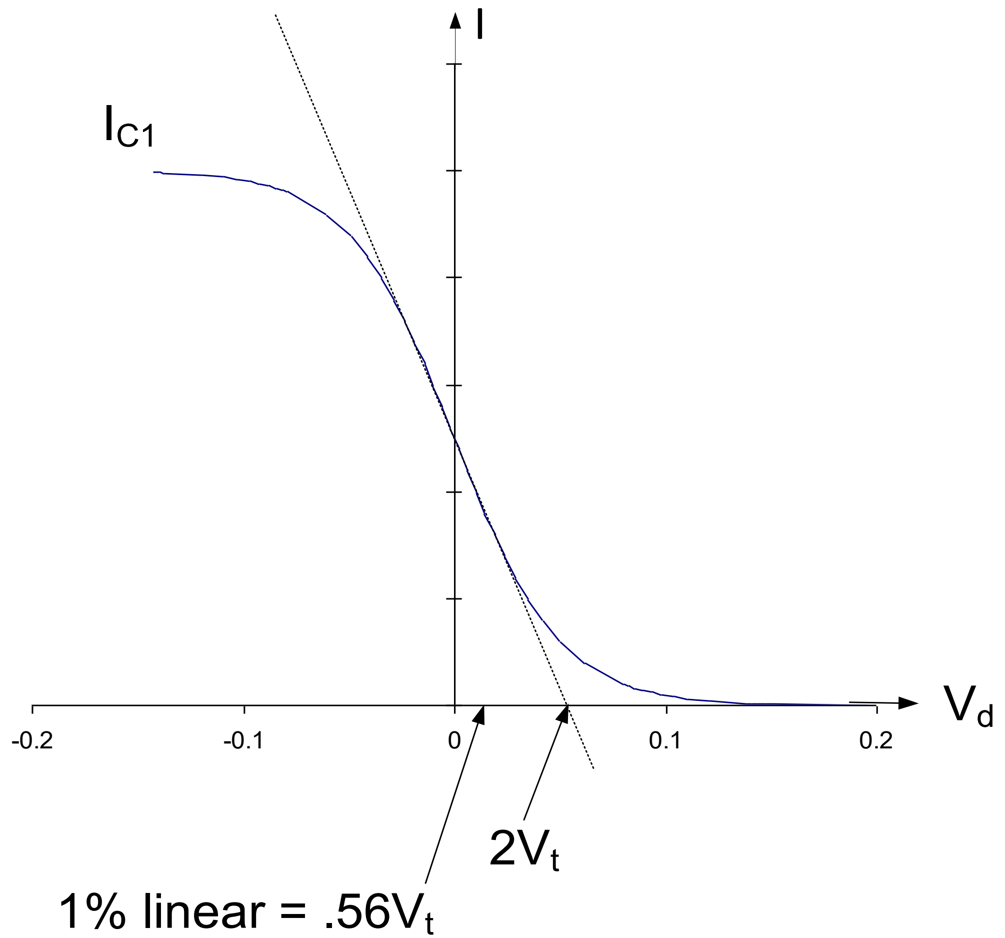
for 1% deviation, V_d=.56V_t

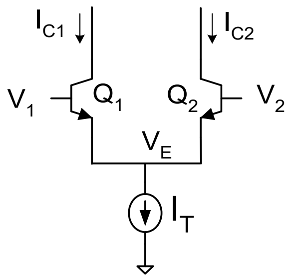
for 0.1% deviation, V_d=.27V_t





Signal Swing and Linearity of Bipolar Differential Pair





Note V_d axis intercept for BJT pair typically much smaller than for MOS pair (V_{EB}) but designer has no control of intercept for BJT pair

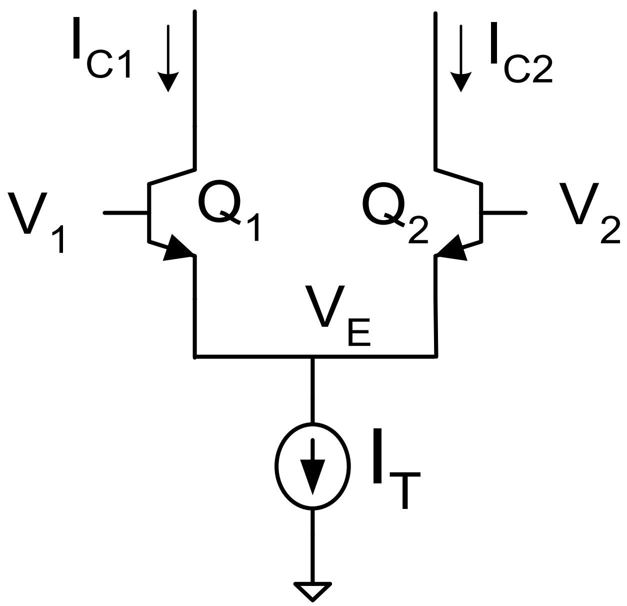
How linear is the amplifier ?

Distortion in the differential pair is another useful metric for characterizing linearity of I_{C1} and I_{c2} with sinusoidal differential excitation

Consider again the differential pair and assume excited differentially with

$$V_2 = \frac{V_d}{2} \qquad V_1 = -\frac{V_d}{2}$$

and assume $V_d = V_m \sin(\omega t)$



$$\mathbf{V_d} = \mathbf{V_2} - \mathbf{V_1}$$

Recall:

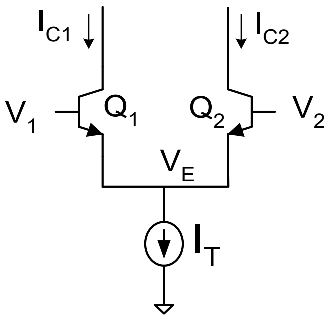
$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

Thus can express as

$$e^{\frac{V_d}{V_t}} = \frac{I_T - I_{C1}}{I_{C1}}$$

$$I_{C1} = I_T\left(1 + e^{\frac{V_d}{V_t}}\right)^{-1}$$

How linear is the amplifier ?



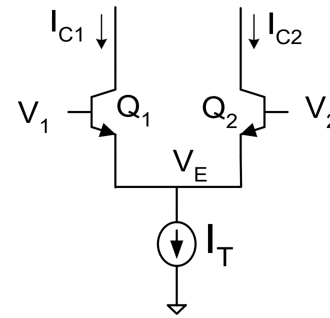
$$I_{C1} = I_T \left(1 + e^{\frac{V_d}{V_t}} \right)^{-1}$$

$V_d = V_m \sin(\omega t)$

Consider a Taylor's Series Expansion

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Bigg|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Bigg|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Bigg|_{V_d=0} V_d^3 \quad + H.O.T$$

How linear is the amplifier ?



$$I_{C1} = I_T \left(1 + e^{\frac{V_d}{V_t}}\right)^{-1} \qquad V_d = V_m \sin(\omega t)$$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 + H.O.T$$

$$\frac{\partial I_{C1}}{\partial V_d} = -\frac{I_T}{V_t}\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}}$$

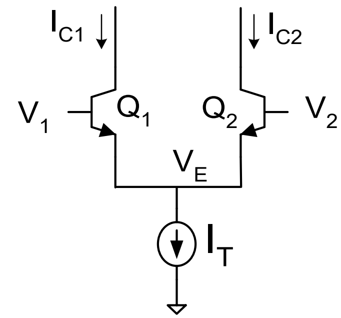
$$\frac{\partial^2 I_{C1}}{\partial V_d^2} = -\frac{I_T}{V_t}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} \frac{1}{V_t} - 2\, e^{\frac{V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} e^{\frac{V_d}{V_t}} \frac{1}{V_t}\right]$$

$$\frac{\partial^2 I_{C1}}{\partial V_d^2} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2\, e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} \frac{1}{V_t} - 2e^{\frac{V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} e^{\frac{V_d}{V_t}} \frac{1}{V_t} + 6\, e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} e^{\frac{V_d}{V_t}} \frac{1}{V_t} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} \frac{2}{V_t}\right]$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3} = -\frac{I_T}{V_t^3}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} + 6e^{\frac{3V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} - 4e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]$$

How linear is the amplifier ?



$V_d = V_m \sin(\omega t)$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 + H.O.T$$

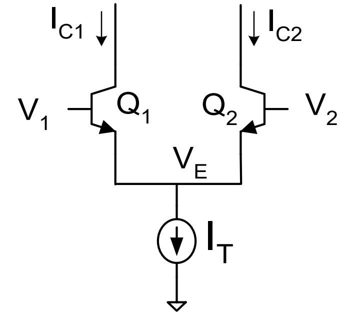
$$\frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} = -\frac{I_T}{V_t}\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}}\Big|_{V_d==0} = -\frac{I_T}{V_t}(2)^{-2} = -\frac{I_T}{4V_t}$$

$$\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2\, e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]\Big|_{V_d=0} = -\frac{I_T}{V_t^2}\left[(2)^{-2} - 2(2)^{-3}\right] = 0$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} = -\frac{I_T}{V_t^3}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} + 6e^{\frac{3V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} - 4e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]\Big|_{V_d=0} = -\frac{I_T}{V_t^3}\left[(2)^{-2} - 2(2)^{-3} + 6(2)^{-4} - 4(2)^{-3}\right] = \frac{I_T}{8V_t^3}$$

$$\frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} = -\frac{I_T}{4V_t} \qquad \frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} = 0 \qquad \frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} = \frac{I_T}{8V_t^3}$$

How linear is the amplifier ?



$V_d = V_m \sin(\omega t)$

$$I_{C1} = I_{C1}\big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\bigg|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\bigg|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\bigg|_{V_d=0} V_d^3 + H.O.T$$

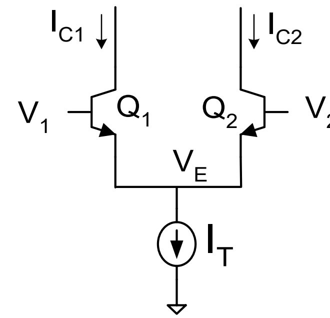
$$\frac{\partial I_{C1}}{\partial V_d}\bigg|_{V_d=0} = -\frac{I_T}{4V_t} \qquad \frac{\partial^2 I_{C1}}{\partial V_d^2}\bigg|_{V_d=0} = 0 \qquad \frac{\partial^3 I_{C1}}{\partial V_d^3}\bigg|_{V_d=0} = \frac{I_T}{8V_t^3}$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_d + \frac{I_T}{48V_t^3} V_d^3$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_m \sin(\omega t) + \frac{I_T}{48V_t^3} V_m^3 \sin^3(\omega t)$$

$$\sin^3(\omega t) = \frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)$$

How linear is the amplifier ?



$V_d = V_m \sin(\omega t)$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 \quad + H.O.T$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_m \sin(\omega t) + \frac{I_T}{48V_t^3} V_m^3 \left[\frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)\right]$$

$$I_{C1} \cong \frac{I_T}{2} + \left[\frac{3I_T}{4 \bullet 48V_t^3} V_m^3 - \frac{I_T}{4V_t} V_m\right]\sin(\omega t) - \frac{I_T}{4 \bullet 48V_t^3} V_m^3 \sin(3\omega t)$$

Thus:

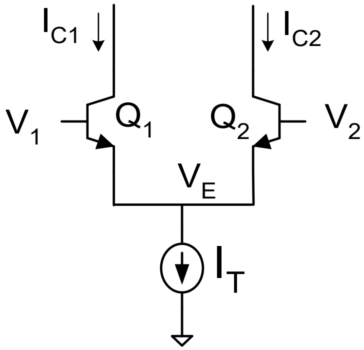
$$THD = 20\log\left(\frac{V_m^2}{\left[48V_t^2 - 3V_m^2\right]}\right)$$

or, equivalently

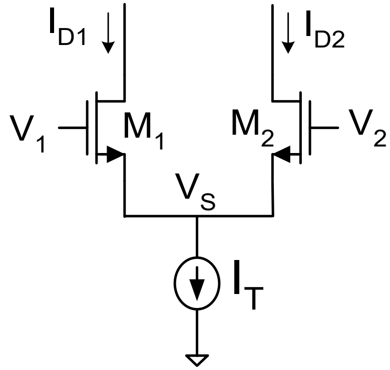
$$THD = -20\log\left(48\left(\frac{V_t}{V_m}\right)^2 - 3\right)$$

V_m/V_t	THD (dB)
2.5	-13.4049
1	-33.0643
0.5	-45.5292
0.25	-57.6732
0.1	-73.6194
0.05	-85.6647
0.025	-97.7069
0.01	-113.625

Comparison of Distortion in BJT and MOSFET Pairs



$$V_d = V_m \sin(\omega t)$$



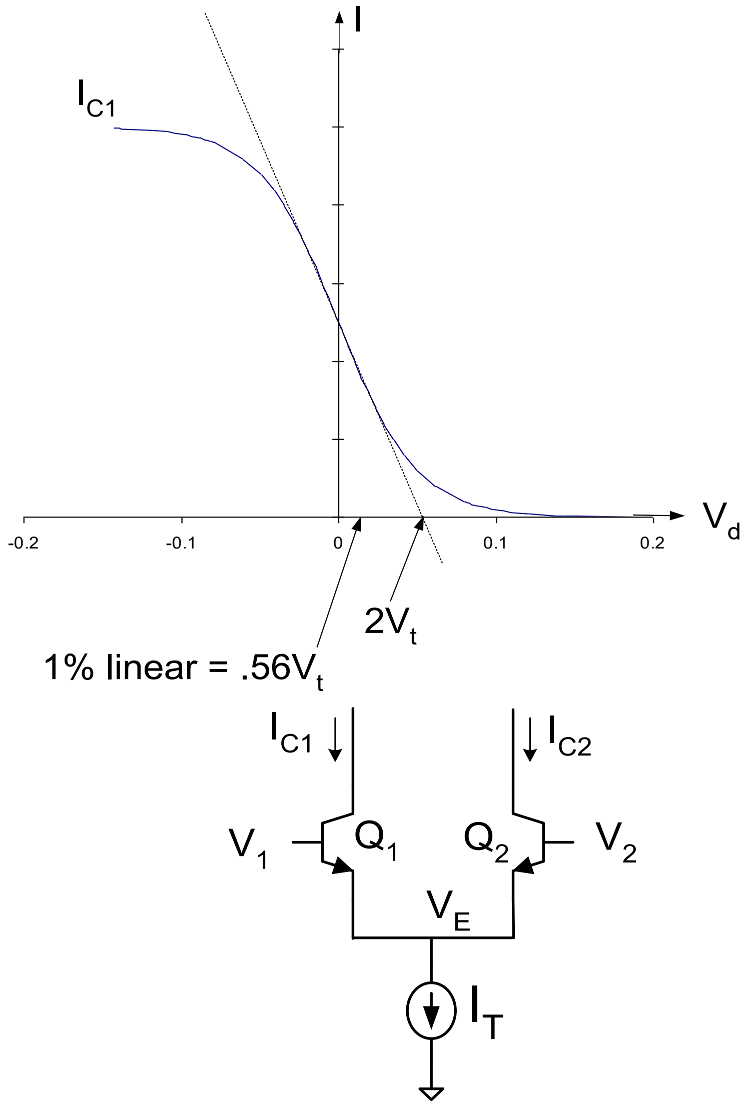
$$THD = -20\log\left(48\left(\frac{V_t}{V_m}\right)^2 - 3\right)$$

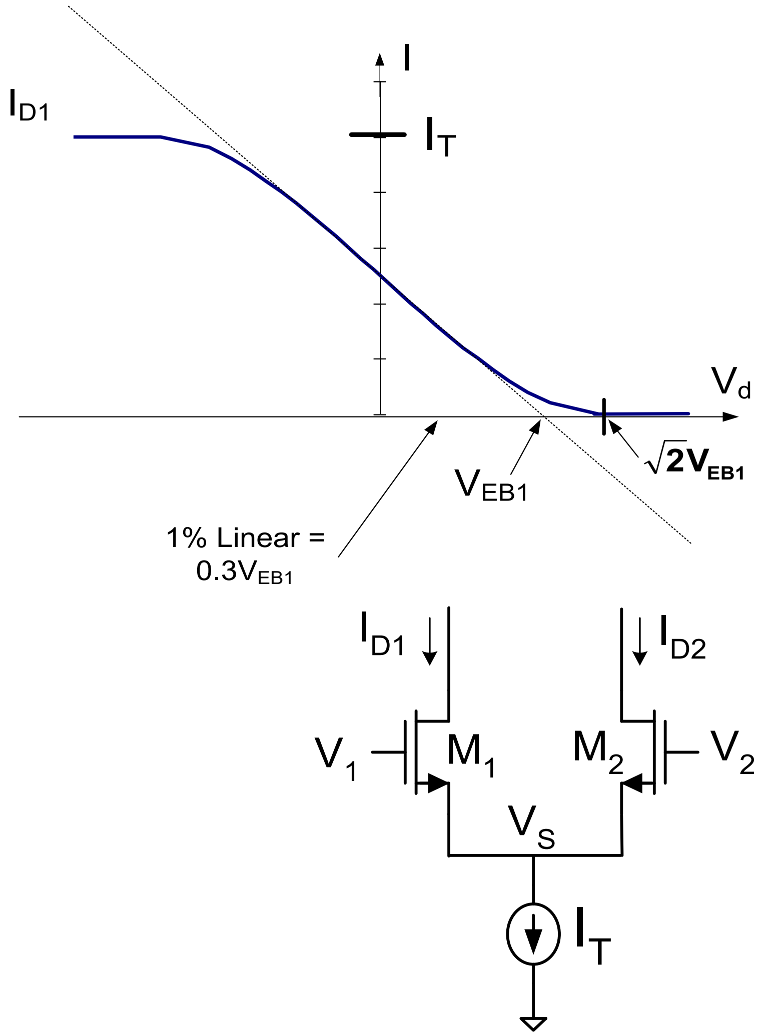
$$THD = -20\log\left(32\left(\frac{V_{EB1}}{V_m}\right)^2 - 3\right)$$

V_m/V_t	THD (dB)
2.5	-13.4049
1	-33.0643
0.5	-45.5292
0.25	-57.6732
0.1	-73.6194
0.05	-85.6647
0.025	-97.7069
0.01	-113.625

V_m/V_{EB1}	THD (dB)
2.5	-6.52672
1	-29.248
0.5	-41.9382
0.25	-54.1344
0.1	-70.0949
0.05	-82.1422
0.025	-94.1849
0.01	-110.103

Linearity and Signal Swing Comparison of Bipolar/MOS Differential Pair





Have completed linearity analysis but must now look at the implications

Stay Safe and Stay Healthy !

End of Lecture 20